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GRAVITY WAVES GENERATED BY SURFACE AND UNDERWATER EXPLOSIONS IN SHALLOW WATER

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A B S T R A C T

Surface gravity waves were produced in shallow water by exploding small charges of Composition C at the surface, at the bottom, and midway between surface and bottom. Weights of 1 oz. to 8 oz. of charge were detonated in water 1 to 2 feet deep. Surface explosions were found to be more effective generators of waves than underwater explosions. Therefore, this report is devoted principally to an analysis of surface bursts.

In the domain of small charges investigated here, the amplitudes and wave lengths associated with surface explosions scale approximately according to the cube root of the charge weight when the distance is scaled by the same factor; the periods and velocities scale according to the sixth root. For surface bursts, the amplitude varies nearly as the inverse first power of the distance. The amplitudes predicted from earlier experiments on the collapse of a cylindrical cavity in water agree reasonably well with those arising from surface explosions. In fact, the efficacy of such explosions in generating gravity waves seems to be best understood in terms of the production and subsequent collapse of a cavity or crater.

A mechanism of wave production by surface explosions is described, in which the dimensions of the cavity are related to those of the gas bubble created by a corresponding underwater explosion.

This mechanism accounts for some of the striking properties of surface bursts, and makes it possible to choose a set of scaling laws for large as well as small explosions. These laws are, however, based on an hypothesis which requires experimental confirmation in the domain of large-scale explosions. A surface burst of the order of 100 tons would help to fill this gap. The prediction of waves produced by a large-scale explosion from those of a small-scale one is based upon the relationship

$$m = \left(\frac{W_1}{W_2}\right)^{\frac{1}{4}} \left(\frac{H_2}{h_2}\right)^{\frac{1}{4}}$$

where m is the scaling factor relating the large amplitudes and wave lengths to the small ones; W_1 is the weight of the large charge, W_2 of the small one; h_2 is the depth of the cavity, and H_2 the water-equivalent depth of the atmosphere in the small-scale experiment. The corresponding scaling factor for periods and velocities is $m^{\frac{1}{2}}$. Estimates are given of the wave heights and periods to be expected from the surface explosion of a nuclear bomb; and the effect of the sea depth is discussed in some detail. For a burst equivalent to 20,000 tons of TNT, a value of 1.6×10^5 ft.^{2} is estimated for the mean product amplitude* x distance in water 200 ft. deep, and 1.1×10^5 ft.^{2} in water 150 ft. deep.

The Appendix discusses certain phenomena in shallow water which appear anomalous at first sight, but which can be understood in terms of the expected behavior of the gas bubble produced by explosions under the conditions of these experiments.

*The term amplitude here denotes crest-to-trough height.

1. Introduction

In a typical underwater explosion, the principal damage to ship structures is inflicted by the underwater shock wave. This should also be true of a nuclear-bomb explosion. However, if the shock wave in the latter case is to be effective, the bomb must be detonated at a depth at least half as great as the lateral distance from the target.[1] This is necessary because the pressure wave is reflected from the free surface of the water as a rarefaction which tends to cancel the positive pressures in the shock wave. Thus, calculations for an explosion equivalent to 20,000 tons of TNT indicate that the radius of lethal damage is approximately 3000 feet, and that for this distance the bomb should be set off at least 1500 feet below the surface. Although lesser depths would obviously suffice for an explosion occurring almost directly under a ship, such distances have to be considered when it is desired to damage more than one ship with a single bomb, or to facilitate the safe delivery of a bomb. On the other hand, depths of the order of 1600 feet do not ordinarily exist in harbors or anchorages, where the greatest concentration of ships is apt to be found.

In the early discussions at the Los Alamos laboratory about the possible use of nuclear bombs against ships, the implicit objective was to inflict serious damage upon a number of vessels at once. This objective was determined by the extraordinary value of the first few nuclear bombs which, at that time, had to be measured in terms of the

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1) LA-545, "Underwater Explosion of a Nuclear Bomb," by John von Neumann and Maurice M. Shapiro, April 8, 1946.

total cost of the project. In view of the obvious military advantages of employing the bomb against an enemy city, its use against ships could be considered seriously only if a single bomb could deliver a major blow against an enemy fleet.*

Because of the rarefaction effect cited above, the shock wave from a nuclear-bomb explosion in shallow water was considered unlikely to cause serious damage to large ships other than those in the immediate vicinity of the burst. The question arose, however, whether the surface gravity waves generated by the explosion might be capable of inflicting enough damage to cripple a significant number of ships in a harbor or anchorage.

It was decided to investigate the process of gravity-wave production in water by means of small-scale experiments. In the first investigation,[2)] carried out in the Spring and Summer of 1944, gravity waves were generated in shallow water by means of "simulated explosions." These consisted of the sudden withdrawal of an immersed cylinder, which resulted in the creation, and subsequent collapse, of a cylindrical cavity in water. By experimenting with cavities of different sizes it was found that the amplitudes of waves from similar cavities scale nearly linearly.

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*With the reduction in the cost of nuclear bombs, once they are made in considerable numbers, this argument breaks down. In fact, as pointed out by J. von Neumann, a single capital ship is a worthwhile target for a nuclear bomb, provided that the industrial effort expended in producing it clearly exceeds the effort required to make a bomb.

2) LA-228. "Gravity Waves Generated By The Creation Of a Cylindrical Cavity in Water," by M. M. Shapiro, February, 1945.

The experiments described in this report, which were performed in the Fall and Winter of 1944-45, are concerned with the wave effects of real explosions in shallow* water. It was desired to know what are the optimum conditions for wave production by explosions and how the wave heights decay with distance from the disturbance. Information was also sought about the periods, velocities, and wave lengths. A further object of this investigation was to determine the scaling laws obeyed by the waves under optimum conditions, with a view to predicting the amplitudes of waves from large scale explosions. Despite the difficulties anticipated in the determination of scaling laws from experiments in shallow water, the latter was chosen deliberately so as to simulate the condition in a harbor.

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*As in Reference 2, the term "shallow" is used in relation to the amount of H.E. detonated. It refers to a depth of water such that the charge in question can create a cavity extending from the surface to the bottom.

2. Apparatus and Procedure

The explosions were set off in a pond 35 feet long, 25 feet wide, and 2.5 feet deep. The size of the pond and, therefore, the range of distances and explosive charges which could be investigated, was limited by the water supply available at Los Alamos. The mud bottom was covered with a few inches of sand and gravel. To prevent the formation of craters by the explosions, a sheet of 1/2-inch armour plate, 5 x 5 feet in size, was laid on the bottom about 15 feet from one end of the pond. Extending longitudinally from this plate, over which the charges were detonated, was another sheet of 1/4-inch steel plate, 8 x 4 feet in size; see Figure 1*. In this way a flat bottom was provided along a radial strip over which the waves were to be measured. Moreover, the complicating effects of having sand, mud and gravel thrown up by the explosion were avoided.

Motion pictures of the wave system against the background of the reference frame described below were obtained with two cameras. A Sept 35 mm camera, running at about fourteen frames per second, was used for the measurement of wave amplitudes; see Figures 3, 4, and 5. When the 35 mm film was projected on the screen of a Recordak Reader the resolution was sufficient to permit a measurement of wave heights accurate to within 5 per cent. For most of the trials, pictures were also taken with a Bell and Howell 16 mm camera at a rate of 64 frames per second. This film provided better time resolution than that

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*The figures appear at the end of this report.

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available with the Sept, thereby permitting reasonably good measurements of periods and velocities. In some instances pictures of the explosion itself were also taken with the Bell and Howell camera; see Figures 2, 24, and 25.

A $\frac{1}{4}$-inch plywood board, 8 feet long and 21 inches high was employed as a reference frame. The board was stiffened longitudinally by two dural angle irons, and supported vertically by steel poles sunk into the bottom of the pond. A set of coordinate lines was painted on the board to serve as a reference system for the measurement of amplitudes and wave lengths; see Figure 3. The near end of the board was placed 5 feet from the explosion, and in some instances 11 feet away. Thus the board extended radially either between 5 and 13 feet or between 11 and 19 feet from the explosion.

Various weights of Composition C* were detonated under a variety of conditions. In a depth of two feet of water, charges were set off at the surface, one foot below the surface, and at the bottom. In a depth of one foot of water, charges were detonated at the surface and at the bottom. Several intermediate depths of water were also used, in order to provide additional evidence concerning the scaling laws. The weights of charge employed were 1 oz., 2 oz., 4 oz., and 8 oz., the last being considered a practical upper limit for the dimensions of the pond. At least two explosions, and usually four or five, were tried under

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*Several of the early charges were pentolite; later it was found convenient to use Composition C. The wave systems produced by the two types of explosive were found to agree within experimental error.

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each set of conditions. In all, more than 180 explosions were set off, and about 150 of these yielded films which were satisfactory for measurement.

3. Surface Explosions vs. Underwater Explosions

Except where otherwise specified, the term "amplitude" will denote the maximum crest-to-trough height of a group of waves. At the distances investigated here, this amplitude was always the sum of the depth of the first trough and the height of the first crest.* The term "range"--except when used in the general sense--will denote a radial distance from the explosive charge, measured in a horizontal plane.

Figures 6 to 10 show the observed wave amplitude as a function of range under a variety of conditions. As a rule, each curve is based upon data from three to five nominally identical explosions. The amplitude was measured at every foot within the range interval shown. The probable errors in the amplitudes from the 8-oz explosions were deduced from the dispersion of the observed values and from the degree

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*Actually, the first disturbance to be propagated is a "swell," which is considerably smaller in amplitude than the first crest. The swell is succeeded by a trough, whose depth is approximately as great as the height of the first crest which follows it. At sufficiently great distances, the succeeding crests are higher than the first one. However, the maximum height of the first crest exceeds that of any subsequent crest.

of precision of the measurements. They are indicated by the vertical error lines in the 8-oz. curve of Figure 6. For the 1-oz., 2-oz., and 4-oz. explosions the probable errors were about 0.7, 0.8, and 0.9 as large, respectively, as those of the 8-oz. amplitudes.*

In Figure 11 a comparison is made of the waves created in water 2 feet deep by an explosion of 1 oz. H.E. when the charge is placed at the surface of the water, at the bottom, and midway between surface and bottom, respectively. The same comparison is made for 2 oz., 4 oz., and 8 oz., respectively, in Figures 12, 13, and 14. It is clear from these graphs that higher waves are produced when the charge is fired at the surface than at the other two depths. Similarly, in water 1 ft. deep, surface bursts create larger waves than explosions set off at the bottom, as can be seen in Figures 15 to 18. The superior efficacy of surface bursts over underwater bursts in generating gravity waves is the most striking result of these experiments.

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*Among the procedures subject to experimental error are: (a) placement of the charge; (b) weighing of the charge; (c) adjustment of the depth of water; in the vicinity of the charge, the depth sometimes differed slightly from its nominal value as a result of the dishing of the armor plate at the bottom; (d) measurement of the projected film. These sources of error were known, and could be minimized. Two other effects may have been present: (e) interaction of the reference board with the waves; and (f) perturbation of the waves by spray and broken water.

4. Scaling Laws for Small Surface Explosions

Since surface bursts appeared distinctly more promising than underwater ones, the former were selected for an investigation of the scaling laws. This problem can be formulated as follows: Let the charge weight be increased by a factor n. Then by what root of n should the depth and range be increased in order to increase the amplitude by the same root? In other words, what is the value of x such that if the depth and range are increased $n^{1/x}$-fold, the amplitude is also increased $n^{1/x}$-fold?

In earlier experiments[2] it had been found that waves from simulated explosions (the collapse of a cylindrical cavity in water) scale nearly linearly, provided that the depths of water are scaled linearly. In the present experiments it was desired to learn whether the same cube-root scaling applies to the waves from real surface explosions. British investigators[3] had concluded from dimensional considerations that, although exact scaling is not possible, approximate scaling should subsist for $x = 4$. It therefore seemed important to determine which of the two values, $x = 3$ or $x = 4$, more nearly fits the facts for the small-scale explosions involved here.

To answer this question, the following experiments were carried out (the results are summarized in Table 1):

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3) Note No. ADM/214/GC.ARB., December, 1944.

(1) <u>Test of Cube-Root Scaling.</u>-- Three comparisons were made under the conditions (a), (b), and (c) of Table 1. These comparisons are shown graphically in Figures 19, 20, and 21. The depths of water employed in each case were in the ratio $d_1/d_2 = n^{1/3}$. Similarly, the wave amplitudes y_1 and y_2 were compared at corresponding ranges $r_1 = n^{1/3} r_2$, and the mean ratio $\overline{y_1/y_2}$ for a series of such distances was computed. Writing

$$\overline{y_1/y_2} = n^{1/x}$$

the reciprocal exponent is then computed from

$$x = \frac{\log n}{\log \overline{y_1/y_2}}$$

The experimental values of x for the three series of explosions (a), (b), and (c) are given in the last column of Table 1. The weighted mean (obtained by weighting each value of x by its reciprocal error) is 3.35 ± 0.4. It can be seen that if the range had been scaled by, say, $n^{1/3.2}$ instead of $n^{1/3}$, the computed value of x would have been smaller, approximately 3.2. This suggests that linear scaling--or, at least, <u>nearly</u> linear scaling-- probably subsists, but it does not exclude the possibility that fourth-root scaling may fit the data nearly as well. For this reason an additional set of comparisons was undertaken.

(2) <u>Test of Fourth-Root Scaling.</u>--In this series, waves were generated under the conditions (d), (e), and (f) of Table 1. The depths of water for a given pair of explosions were in the ratio $d_1/d_2 = n^{\frac{1}{4}}$, and

Table 1. Scaling of Waves from Surface Explosions*

	Series	Explosion 1 W_1	Explosion 1 d_1	Explosion 2 W_2	Explosion 2 d_2	n	$p = n^{1/3}$	y_1/y_2	$x = \frac{\log n}{\log y_1/y_2}$
Cube Root Test	(a)	8	24	1	12	8	2.00	1.85	3.38 ±0.3
	(b)	8	24	2	15	4	1.59	1.45	3.73 ±0.6
	(c)	4	24	1	15	4	1.59	1.56	3.12 ±0.3
							$p = n^{1/4}$	Weighted Mean →	3.35 ±0.4
Fourth Root Test	(d)	8	24	1	14.3	8	1.68	2.00	3.00 ±0.5
	(e)	8	24	2	17	4	1.41	1.48	3.54 ±0.5
	(f)	4	24	1	17	4	1.41	1.74	2.50 ±0.5
								Weighted Mean	3.01 ± 0.4

*W = weight of Comp. C in oz.
d = depth of water in inches
$n = W_1/W_2$
y = Amplitude of wave at range r.
y_1 = Amplitude at range $r_1 = pr_2$

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amplitude ratios were obtained for distances r_1, r_2 such that $r_1 = n^{\frac{1}{4}} r_2$. Again the reciprocal exponent x was computed in each case. Its mean value turned out to be 3.01 ± 0.4. Since the value $x = 4$ lies well outside the range of experimental error, it appears that our data is inconsistent with a $W^{\frac{1}{4}}$- scaling law. Thus, in the domain of small-scale surface explosions investigated here, the wave amplitudes scale much more nearly according to a cube-root law than a fourth-root law. The former can be expressed as follows:

$$\frac{y_1}{y_2} = \frac{d_1}{d_2} = \left(\frac{W_1}{W_2}\right)^{1/3} = \frac{r_1}{r_2}$$

In other words, if the depth of water, the linear dimensions of the charge, and the range are increased by a given factor, the amplitude is increased by the same factor.

This conclusion is based upon experiments with relatively small charges, and should not be applied uncritically to large-scale explosions. There is, in fact, reason to believe that for explosions involving charges of the order of hundreds of tons or larger, the amplitudes scale according to $W^{\frac{1}{4}}$. The argument for this is given in Section 9.

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5. Further Evidence for Cube-Root Scaling of Small Explosions

There is another line of evidence in facor of cube-root scaling for small surface explosions. In earlier experiments[2)] performed in a small laboratory tank, the wave effects of an explosion were simulated by the sudden withdrawal of a cylinder immersed in shallow water. This produced a cavity whose collapse generated a wave system. The dimensions of the cavity could be varied by changing the diameter of the cylinder and the depth of the water. In this way it was observed that the amplitudes of the waves from geometrically similar cavities scale nearly linearly. The maximum cavity diameter in those experiments was 1 ft., and the maximum depth, 6 inches. The results were expressed non-dimensionally, however, and if the same scaling law obtained for larger cavities, then these results also predict the wave amplitudes from such cavities.

In order to compare these predictions with the amplitudes from real explosions, let us assume that, to a first approximation, the crater produced by a surface explosion in shallow water may be treated as a cylindrical cavity. We assume further that the depth of this cavity equals (at least for the 4-oz. and 8-oz. charges) the depth of the water, and that the diameter is approximately equal to the maximum diameter of the gas bubble formed by an underwater explosion. Using the formula for R_m, the maximum radius, which is given in the Appendix, this diameter is found to be about 6.1 ft. for our 4-oz.

explosions, and 7.7 ft. for our 8-oz. explosions.* Evidence that the diameters our our explosion craters actually approximated these dimensions was obtained from cinematic pictures such as those in Figures 24 and 25, by measuring the diameter of the spray column near its base. The most reliable data in the present experiments were obtained with the 4- and 8-oz. charges in 2 ft. of water, and we shall therefore compare these with the results of the simulated explosions.

Now, in the artificial-cavity experiments, a significant parameter was found to be R, the ratio of diameter to depth of cavity; and most of the results obtained were for integral values of R. If, therefore, we adopt for simplicity the approximate value of 8 ft. for the diameter of the 8-oz. crater and 6 ft. for the 4-oz. crater, we shall have two cavities with integral R for direct comparison with the earlier predictions: $R = 8/2 = 4$ for 8 oz., and $R = 6/2 = 3$ for 4 oz. The two comparisons are shown in Figure 22.** The upper graph compares

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*These values are somewhat larger than those in Table 10 (Appendix) because the pressure at the surface is equivalent to 25 feet of water at Los Alamos, whereas the values of R_m in Table 10 are based on a total pressure equivalent to 26 feet.

**The dashed curves were obtained from Figure 13, Reference 2, the only difference being that in the latter, the coordinates are non-dimensional, whereas here they are given in absolute units. The plot is logarithmic since the wave amplitudes decrease with range according to a power law.

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the predicted wave amplitudes for the collapse of a cavity 8 ft. in diameter and 2 ft. deep (dashed curve) with the actually observed amplitudes from a surface explosion of 8 oz. Comp. C in 2 ft. of water (solid curve). The lower graph makes the same comparison between a 6 x 2 ft. cavity and a 4-oz. explosion.

One would hardly expect very close agreement between the observed amplitudes and those predicted by the cavity-collapse experiments, considering that the assumption of cylindrical shape for the explosion craters is merely a convenient idealization. In view of this, Figure 22 shows remarkable agreement not only in the absolute value of the amplitudes, but also in their rate of decay with distance. This provides additional evidence for cube-root scaling, since the predicted amplitudes (dashed curves) were based on the assumption that our explosion waves scale linearly.

This agreement also suggests that the character of the waves generated by surface explosions in shallow water can be understood by attributing them to the creation and sudden collapse of a cavity extending to the bottom. The implications of this mechanism of wave production for the scaling laws which govern very large explosions will be discussed below in Section 9.

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6. Variation of Wave Amplitude with Distance in Surface Explosions

The maximum range, about 18 ft., at which waves were measured in these experiments was determined by the size of the artificial pond. It is possible, however, to make reasonable estimates of the amplitudes at larger distances from the explosion. There is evidence from at least two sources[4,5] that at distances between 5 and 50 times the depth, the amplitude varies nearly as the inverse first power of the distance.

As before, let $y =$ amplitude, and $r =$ range. Then in our surface explosions of 8 oz. in 1 ft. of water, the product yr decreases slowly with distance, whereas in 2 ft. of water, it increases slowly. However, the mean values $\overline{yr} = 3.58$ and $\overline{yr} = 7.36$ ft.^{2}, respectively, fit the observed data fairly well. In view of the evidence cited above for an r^{-1} decay at greater distances, it would seem to be a reasonable method of extropolating the amplitude-range curves to make the product yr approach the observed $\overline{yr}$ at large distances. This extrapolation is employed in a subsequent section, in which the wave amplitudes from a nuclear-bomb explosion are estimated.

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4) New Zealand Project "Seal", Note of 15 June 1945, issued by the Department of Science and Industrial Research, Dominion of New Zealand; also see subsequent "Seal" reports. These experiments in New Zealand were carried on simultaneously with and independently of those at Los Alamos.

5) BuShips Scientific Memorandum No. 114, dated 1 November 1944, by G. A. Ross.

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7. <u>Velocities, Periods, and Wave Lengths</u>

The velocity of the first, and highest, crest produced by surface explosions in 2 ft. of water was measured directly from the motion pictures for several bursts of each of 1-, 2-, 4-, and 8-oz. charges. The period of the leading wave--the one containing the first crest--was obtained by measuring the time between the arrival of the first and second troughs at a given distance. In view of the evidence for cube-root scaling of the amplitudes from small surface explosions, it was expected that the wave lengths would likewise scale linearly at corresponding ranges. Accordingly, the various periods were measured at distances approximately proportional to $W^{1/3}$. It was convenient to use the distances 8, 10, 13, and 16 ft., respectively, for the four charge weights. Similarly, the velocity was measured over a range interval whose midpoint was the same corresponding distance, <u>viz</u>. from 6 to 10 ft., 8 to 12 ft., 11 to 15 ft., and 14 to 18 ft., respectively.

The results are listed in Table 2. Each set of values is based on data from four or five explosions. The wave length was obtained by multiplying the velocity by the period.

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Table 2. Observed Values of Velocities, Periods, and Wave Lengths

Weight W (oz)	Velocity V (ft/sec)	Period T (sec)	Wave Length $\lambda = VT$ (ft)
1	4.4±0.3	1.04±0.6	4.6±0.5
2	5.0±0.5	1.06±0.6	5.3±0.8
4	5.4±0.4	1.18±0.6	6.4±0.7
8	6.0±0.5	1.39±0.6	8.3±1.0

For gravity waves, the linear dimensions are proportional to gt^2. Hence, if amplitudes scale as $W^{1/3}$ for small explosions, we should expect time intervals to scale as $W^{1/6}$. It is easy to see dimensionally that the latter should also be true of velocities. Thus, wave lengths should scale as $W^{1/3}$. That the data in Table 2 are consistent with these scaling laws is shown in Table 3. Here, the velocities and periods of the 1-, 2-, and 4-oz. waves are scaled up to 8 oz. by the sixth root of the appropriate charge ratio. The calculated velocities agree very well with that observed for 8 oz., and the periods agree reasonably well. In view of this, the averages of these velocities and periods, as well as the product of the averages (the mean wave length) are probably somewhat more reliable than the

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Table 3. Velocities and Periods Scaled up to 8-oz. Explosion, Using a $W^{1/6}$ Law

Weight W (oz)	$(8/W)^{1/6}$	$(8/W)^{1/6}\,V$ (ft/sec)	$(8/W)^{1/6}\,T$ (sec)
1	1.414	6.2	1.47
2	1.26	6.3	1.34
4	1.123	6.1	1.33
8	1.00	6.0	1.39
	Averages:	$\bar{V} = 6.15\ \frac{ft}{sec}$	$\bar{T} = 1.38$ sec
		$\bar{\lambda} = \overline{VT}$	= 8.5 ft.

values based on the 8-oz. measurements alone.* (Unweighted averages are considered adequate because the relative errors in the four sets of values in Table 2 are much alike in magnitude.) It is interesting to note, however, that all three averages agree, well within the experimental errors, with the direct observations on 8-oz. bursts.

A set of <u>probable</u> values of velocities, periods, and wave lengths for bursts of each size is given in Table 4. These values were obtained by scaling down from the cited averages for an 8-oz.

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*For this reason it is these average values which are used below (Section 10) in estimating the periods and velocities of the waves from a nuclear-bomb explosion.

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Table 4. Probable Values of Velocities, Periods, and Wave Lengths

Weight W (oz)	Velocity V (ft/sec)	Period T (sec)	Wave Length $\lambda = VT$ (ft)
1	4.35 ± 0.4	0.98 ± 0.06	4.3 ± 0.7
2	4.9 ± 0.4	1.10 ± 0.06	5.4 ± 0.7
4	5.5 ± 0.4	1.23 ± 0.06	6.8 ± 0.7
8	6.15 ± 0.4	1.38 ± 0.06	8.5 ± 0.7

explosion (cf. Table 3), using again the $W^{1/6}$ scaling law for the velocities and periods.

Finally, it should be said that theoretically, one expects the velocities and wave lengths associated with a given burst to increase with distance from the explosion.[6] However, the small size of the pond employed in these experiments permitted only a limited range of distances, insufficient for an investigation of the rate of this increase.

6) "Gravity Waves in Water Caused by Explosions", by W. G. Penney, LA Report 215, February 5, 1945.

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8. "Anomalous" Results

Some of the results obtained in these experiments appear rather curious at first sight. For example, when detonated at the bottom in 2 ft. of water, 2 oz. of H.E. generates waves larger than those from 4 oz., and approximately as large as those from 8 oz. This, and other apparently anomalous results, can be explained by considering the behavior of the gas globe produced by an underwater explosion, together with certain characteristics of the production of water waves by explosions. This subject is treated in the Appendix.

9. Mechanism of Wave Production by Surface Explosions

We have presented evidence that the wave amplitudes scale approximately as the cube root of the explosive weight for surface bursts of small charges. Clearly, we cannot assume that this scaling law extends up to very large explosions.

Before attempting to predict the magnitude of waves from large-scale surface explosions, it is necessary to have some notion of the process whereby these waves are created. The mechanism of gravity-wave production by underwater explosions has been qualitavely described in an earlier report (cf. Introduction, Reference 2), and W. G. Penney,[6)] has given a theory for the generation of such waves which is in substantial agreement with experiment. Unfortunately, no

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satisfactory theory exists for the creation of waves by <u>surface</u> explosions. A mechanism proposed by Penney in the same paper[6] is based upon the initial delivery to the surface of a downward impulse over a certain area. A scaling law deduced from this theory states that wave heights at corresponding distances are proportional to the sixth root of the charge weight. This conclusion is in sharp disagreement with our experiments (*cf*. Section 4).

It should be mentioned that Penney's theory applies to deep water, whereas our experiments were performed in shallow water. However, one would hardly expect this difference to result in a change from a sixth-root law to a cube-root law. In fact there is evidence* that for depths less than about 0.8 times the wave length, the wave amplitudes decrease with decreasing depth; so we would expect our experimental values to be smaller, rather than larger, than those predicted for deep water. Moreover, experiments in both deep and shallow water with larger charges ($\frac{1}{2}$ lb. to 300 lbs.) by the New Zealand "Seal" Project[7] yielded a scaling law between $W^{1/4}$ and $W^{1/3}$, and "over the limits of distance considered, the cube root law...is a closer fit than the fourth-root law..." It is clear, then, that Penney's theory of wave production

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*Reference 4, Figure 2.

7) Interim Report of the New Zealand Seal Project, summarizing results up to 31 October 1944. Auckland, N. Z., 1945.

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by surface explosions leads to predicted amplitudes considerably smaller than those actually observed.

While no attempt at a rigorous theory is made here, we shall describe an hypothesis which may lead to an adequate theory. This hypothesis will explain:

(1) Why surface explosions are more effective wave-generators than underwater ones.

(2) Why the wave amplitudes from small explosions scale as the cube root of the charge weight.

(3) How a well-defined "fountain," or column of spray, such as that in Figure 24, can be thrown up vertically by a surface explosion.

After showing how the hypothesis accounts for the foregoing facts, we shall deduce its implications for the scaling of large explosions.

Assume that a surface burst creates a cavity in the water, whose diameter is of the order of the maximum diameter of the gas globe from a corresponding underwater explosion.* By "corresponding" is meant an equal charge fired at a depth such that the gas bubble will just break the surface at its first maximum expansion (i.e.

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*Evidence that this is a good approximation is cited in Section 5.

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at Penney's optimum depth for wave production by underwater explosions[6]). The explosion produces the cavity mainly by blowing water outward and upward,* rather than by pushing back a considerable mass of surrounding water; i. e. most of the momentun transferred to the water by the explosive is imparted locally, to water in or near the cavity, and a relatively small amount of momentum is imparted to the large mass of surrounding water.

A diagram illustrating the nature of the flow inside the cavity at two stages of its formation is given in Figure 23. In the first stage the jets emerging at the periphery of the cavity are projected outward as well as upward. In the second stage the sidewise component in the peripheral spray is smaller; and, in the final stage (not shown), this spray is thrown directly upward. A comparison of the earliest explosion photographs with the later ones in the cinematic sequences of Figures 24 and 25 lends plausibility to these ideas. It is thus possible to understand how a surface explosion can lead to the formation of a "fountain" such as that shown in Figures 24 and 25.

If the energy available for cavity-formation is largely expended locally by the "scouring" action described above, this explains the superior efficacy of surface over underwater explosions in producing waves. For in the latter case a considerable amount of the explosion

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* The writer is indebted to Dr. F. Reines for demonstrating that a mechanism of this kind appears to operate in a cavity formed by blowing a jet of gas downward against a water surface.

energy goes into the radial mass motion of the water surrounding the gas bubble.

Another clue to the mechanism of wave production by surface explosions may be found in Figure 24. In the last few frames of this figure there appears a thin central jet of water. Although no measurements are available on the total height of this jet, it was confirmed by visual observation of scores of surface explosions that the jet was thrown to a height several times that of the main body of spray. There is good evidence that this jet is produced by the collapse of the cavity*, and it is reasonable to regard the great height of the jet as an indication of the violence of collapse. The gases inside the cavity formed by

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*In the original prints of Fig. 24, the jet can first be seen in the second frame of the film strip at the right, i.e. at about 0.125 second after the explosion. Now, we can calculate the period of the first pulsation of a gas bubble produced under the conditions of Fig. 24, applying the formula (*cf*. Reference 10, Appendix):

$$T = 1.135\ \rho^{1/2} p^{-5/6} E^{1/3}$$

ρ is the density of the water in gm/cm^3,
p is the hydrostatic pressure in dynes/cm^2,
and E is 0.4 of the total energy in ergs released by the explosion.

Taking for p the equivalent of 25 ft. of water (one Los Alamos atmosphere), and for the energy liberated by Composition C, 1300 cal./gm, we obtain $T = 0.123$ second for the time of the first minimum. Since, within experimental error, the jet arises at the same time, there seems little doubt that it is produced by the collapse of the cavity.

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a surface explosion are not nearly as well confined as are those in an underwater explosion. Hence, the collapse should be more complete and violent than the contraction of an underwater bubble. Because of the cushioning effect of the gases inside, the latter does not collapse completely at the end of its first pulsation, but is reduced only to approximately one half of its maximum diameter. This difference is perhaps an additional reason for the greater amplitude of waves from surface explosions.

Our hypothesis assumes that the dimensions of the cavity produced by a surface explosion are of the same order as those of the bubble cavity from a corresponding underwater explosion. Penney has deduced the scaling laws for the latter case*, and since our initial condition may likewise be taken as a surface displacement, the same scaling laws should apply. The arguement goes as follows: The energy E which goes into the expansion of the bubble is given approximately by

$$E = VP$$

where V is the volume of the bubble at its maximum size and P is the total pressure (including atmospheric) at the center of the cavity.

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*Reference 6, pp. 5-6, see "Case I. Initial Surface Displacement."

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Let h = maximum bubble radius (or, in our case, a linear dimension of the cavity),

H = depth of water corresponding to one atmosphere,

W = weight of charge.

Then $E \propto W$; $V \propto h^3$; and $P \propto (h + H)$.

Therefore, $$W \propto h^3(h + H)^{*} \qquad (9.1)$$

Now, in comparing two explosions, let us assign the subscript 1 to the parameters of the larger explosion, and subscript 2 to those of the smaller one.

From (9.1), we have: $$\frac{W_1}{W_2} = \frac{h_1^3}{h_2^3} \frac{(h_1 + H_1)}{(h_2 + H_2)} \qquad (9.2)$$

(Note that H_1 may differ from H_2, e.g. in a comparison between an explosion at Los Alamos and one at sea level).

Let $h_1/h_2 \equiv m$.

From a generalization of the fundamental solution for cylindrically expanding waves, Penney obtains the following scaling laws:

$$\frac{y_1}{y_2} = \frac{d_1}{d_2} = \frac{r_1}{r_2} = \frac{t_1^2}{t_2^2} = m \qquad (9.3)$$

$$y_1 (m\, r_2, \sqrt{m}\, t_2) = m\, y_2 (r_2, t_2) \qquad (9.3a)$$

where d = depth of water
r = range
t = time after explosion

$y(r,t)$ = amplitude at range r and time t: Since the scaling laws are determined by the value of m, we need only solve Eq. (9.2) for this ratio.

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* Rigorously one should, in the shallow-water case, distinguish between two kinds of h: the lateral dimension of the cavity, R, and the depth, D; thus, h^3 should be R^2D. The scaling laws, however, are [redacted] the same if h^3 is used. In Eq.(9.6), below, h_2 must be the depth, and it is so interpreted in Sec. 10. It should be mentioned that in our experiments, where D differed at most by a factor 4 from the "optimum depth," R was determined almost entirely by W, and changed only slightly with D.

[redacted]

Consider three cases:

(1) Comparison of two small-scale explosions. If, as in our experiments, h_1 and h_2 are of the order of 1 or 2 ft.,

$$H_1 \gg h_1 \quad \text{and} \quad H_2 \gg h_2$$

and Eq. (9.2) reduces to

$$\frac{h_1^3}{h_2^3} = \frac{W_1}{W_2}\left(\frac{H_2}{H_1}\right)$$

If, moreover, both experiments were performed under the same atmospheric pressure, then $H_1 = H_2$, and

$$m = \frac{h_1}{h_2} = \left(\frac{W_1}{W_2}\right)^{1/3} \qquad (9.4)$$

I.e., the amplitudes, distances and depths scale according to the cube root of the charge ratio.

(2) Comparison of two large-scale explosions. If both explosions are of hundreds of tons or more, then the cavity dimensions are of the order of hundreds of feet. Thus

$$h_1 \gg H_1 \quad \text{and} \quad h_2 \gg H_2.$$

Eq. (9.2) reduces to

$$\frac{W_1}{W_2} = \frac{h_1^4}{h_2^4}\ ; \text{ and}$$

$$m = \frac{h_1}{h_2} = \left(\frac{W_1}{W_2}\right)^{1/4} \qquad (9.5)$$

In this case the various linear dimensions scale as the fourth root of the charge ratio.

[redacted]

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(3) <u>Prediction of very large-scale explosions from small-scale results</u>. Here again, $h_1 \gg H_1$, but

$$h_2 \ll H_2.$$

Therefore, eq. (2) reduces to

$$\frac{W_1}{W_2} = \frac{h_1^3}{h_2^3}\left(\frac{h_1}{H_2}\right)$$

$$= \frac{h_1^4}{h_2^4}\left(\frac{h_2}{H_2}\right)$$

Therefore,

$$m = \frac{h_1}{h_2} = \left(\frac{W_1}{W_2}\right)^{1/4}\left(\frac{H_2}{h_2}\right)^{1/4} \qquad (9.6)$$

Here, the scaling factor which relates the amplitudes, distances, and depths of the two explosions depends not only on the charge ratio, but also upon the ratio H_2/h_2 which characterizes the smaller explosion.

It is worth emphasizing that the underlying reason why the scaling law which subsists for small charges does not apply to large ones is the existence of a (relatively) fixed atmospheric pressure. In order to preserve similitude, the latter would have to be varied in proportion to the depth of water. Instead, it is much larger than that depth for small-scale bursts and much smaller for explosions on the scale of a nuclear bomb. This, as we have seen,

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[redacted]

leads respectively to a cube-root scaling law in the first case, and to a fourth-root law in the second.

Theoretically there is good reason[*] to expect that if the air pressure above the water could be suitably reduced in small-scale experiments (so that $H \propto h$ in Eq. (9.2), $W^{\frac{1}{4}}$ scaling would apply--at least approximately--in the entire range from small to large charges. G. I. Taylor and his collaborators[7a)] have studied underwater bubble phenomena with apparatus of this kind. For wave investigations, a pressure-sealed tank much larger than the one used by Taylor and a liquid other than water, with a low vapor tension, would be required. Even with a large tank (say, 10 x 8 x 5 ft.), the scale of the explosions and waves would necessarily be considerably smaller than that of our pond. Moreover, the entire experimental arrangement would obviously be far more elaborate. However, these disadvantages would probably be offset by the increased confidence with which large-scale wave phenomena could be predicted.

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*This can be shown by applying Buckingham's π-Theorem to our situation; cf. Phys. Rev., 4 (II), 345 (1914). The relevant variables are the energy released by the charge, the hydrostatic pressure, the density of the water, the acceleration of gravity, the time after the explosion, the linear dimensions, e.g. of the cavity or of the waves, and the efficiency of conversion of chemical energy into energy of wave motion.

7a) British Report S.W.29, Undex 13.

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10. Surface Explosion of a Nuclear Bomb.

If we accept as a working hypothesis the mechanism of surface explosions described in the preceding section, and the scaling laws based on this hypothesis, we are in a position to estimate the magnitude of the waves from a surface explosion of a nuclear bomb.

For this purpose we choose the results of the 8-oz. surface bursts (Figures 6 and 9) in 1 and 2 feet of water, respectively, because the relative errors in the measured amplitudes were smallest for these charges. From Eq. (9.6), the scaling factor m which relates the linear dimensions of very large-scale explosions to those of small ones is given by:

$$m = \frac{h_1}{h_2} = \left(\frac{W_1}{W_2}\right)^{\frac{1}{4}} \left(\frac{H_2}{h_2}\right)^{\frac{1}{4}}$$

At Los Alamos, $H_2 = 25$ ft.

Table 5 lists values of m and h_1 for explosions involving TNT equivalents of 10, 20, and 50 kilotons. Figure 26 gives estimates, for these nuclear-bomb efficiencies, of the crest-to-trough wave height as a function of the range. Not only the amplitude and range, but also the depth is scaled up by the factor

[redacted]

Table 5. Scaling From Small to Large Surface Explosions.

	W_1 (kilotons)	$\left(\frac{W_1}{W_2}\right)^{1/4}$	$\left(\frac{H_2}{h_2}\right)^{1/4}$	m	h_1 (feet)
$W_2 = 8$ oz. $h_2 = 1$ ft.	10	79.5	2.236	178	178
	20	94.6		212	212
	50	118.9		266	266
$W_2 = 8$ oz. $h_2 = 2$ ft.	10	79.5	1.88	149	298
	20	94.6		178	356
	50	118.9		224	448

m from the 8-oz. curve in Figure 9, where the depth was 1 ft. This explains the depths (rounded off to the nearest 5 ft.) given in the figure. Similarly the curves in Figure 27 are scaled up from the 8-oz. curve in Figure 6, where the depth was 2 ft. The crest amplitudes, measured from still-water level, would be approximately one-half of the crest-to-trough values in Figures 26 and 27.*

The solid part of these curves is scaled up from the experimental data in Figures 6 and 9. The broken-line portion of the curves is an extrapolation to larger ranges, which fits the

* Note added in print: Our amplitude values should be reduced by approximately 10 per cent in order to take account of the difference in energy between Comp. C and TNT.

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formula $$yr = km^2 \qquad (10.1)$$

Here y, r, and m have the same significance as before, and k is the observed value of $\overline{yr}$ for the small-scale prototypes, i.e. $k = 3.58$ ft.^{2} for 8 oz. in 1 ft. of water, and 7.36 ft.^{2} in 2 ft. of water. The factor m^2 arises from the fact that both the amplitude and range are scaled up by the factor m. It will be recalled (cf. Section 6) that the small-scale curves are fairly well represented by $yr = k$, even for small ranges. Similarly, Eq. (10.1) is a reasonably good representation of the curves in Figures 26 and 27. The values of km^2 are listed in Table 6.

For large-scale waves, evidence that the amplitude decays approximately as r^{-1}, especially at large distances, was obtained by Japanese observations of the "tunami" waves which originate in disturbances of the sea produced by earthquakes[7)].

Table 6. The Product yr* for Waves from a Nuclear Bomb at the Surface.

W kilotons	Depth of Water (ft.)	$\overline{yr} = km^2$ (ft^2)
10	180	1.13×10^5
20	210	1.61 "
50	265	2.53 "
10	300	1.63 "
20	355	2.33 "
50	450	3.69 "

*y = crest-to-trough amplitude;
r = range

[redacted]

In estimating the velocities and periods of the waves from a nuclear bomb explosion, we recall* that a scaling factor m for linear dimensions implies a scaling factor $m^{\frac{1}{2}}$ for velocities and periods. If we scale up from the average results listed in Table 3 for 8 oz. in 2 ft. of water, we arrive at the values of V, T, and λ in Table 7. The values of m are the same as those in the second half of Table 5. The velocity is that of the first crest, as in the small-scale experiments; the period is the time interval between the arrival of the first and second troughs (respectively preceding and following the first crest) at the distance r.

It should be noted that all of these estimates of velocities, periods, and wave lengths are for a mean distance r from the explosion. In the 8-oz. burst measurements, this distance was 16 ft., and the values of r in Table 7 were scaled up from this distance by the factor m. As mentioned earlier*, the velocity and wave length are theoretically expected to increase with distance from the source.

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*See Section 7.

Table 7. Velocities, Periods, and Wave Lengths of Waves from Nuclear-Bomb Explosions at the Surface*.

TNT-Equivalent (kilotons)	Depth (ft.)	m	$m^{\frac{1}{2}}$	V (ft/sec)	T (sec)	$\lambda = VT$ (ft)	r (ft)
10	300	149	12.2	75	16.8	1260	2400
20	355	178	13.3	82	18.4	1510	2850
50	450	224	15.0	92	20.7	1900	3600

*Scaled up from 8-oz. bursts in 2 ft. of water; cf. Section 7, and Tables 3 and 4.

11. Effect of Sea Depth on the Waves from Surface Explosions.

The estimates of wave amplitudes in Section 10 (Figures 26 and 27) were made for water of a given depth, scaled up in each case by a factor m from a small-scale experimental depth. It is of interest to estimate the effects upon amplitude of

(a) Shallower water: this would apply to most harbors and anchorages; and

(b) Deep water: the open sea.

In our small-scale experiments only two depths, 1 ft. and 2 ft., were employed. For bursts of 8 oz. in these respective depths, we found values (cf. Section 6) of 3.58 ft.^{2} and 7.36 ft.^{2} for the

[redacted]

mean products $\overline{yr}$ of amplitude and range. While it is hardly possible to generalize on the basis of two depths, these figures at least suggest that—for depths small compared with the wave length—$\overline{yr}$ is roughly proportional to the depth.

Fortunately, the New Zealand Seal project has investigated the variation of $\overline{yr}$ with depth. Their results are given non-dimensionally* in terms of $y_o r_o$—the product yr in deep water—and λ_o, the wave length in deep water. Specifically, $yr/y_o r_o$ is plotted against h/λ_o, where h is the actual depth. An examination of the Seal curve shows it to be nearly linear for $h < 0.2\,\lambda_o$. At these depths it conforms approximately to the equation

$$\frac{yr}{y_o r_o} = 2.4\,\frac{h}{\lambda_o} - 0.01 \qquad (h < 0.2\,\lambda_o) \qquad (11.1)$$

Let $\overline{y_1 r_1}$ be the mean product for a depth h_1. To evaluate $\overline{yr}$, corresponding to some shallower depth, h, we can write

$$\frac{\overline{yr}}{\overline{y_1 r_1}} = \frac{2.4\,(h/\lambda_o) - 0.01}{2.4\,(h_1/\lambda_o) - 0.01} \qquad \begin{matrix}(h < 0.2\,\lambda_o)\\ (h_1 < 0.2\,\lambda_o)\end{matrix} \qquad (11.2)$$

For our purposes we may simplify Eq. (11.2) to the approximate form**

$$\frac{\overline{yr}}{\overline{y_1 r_1}} \doteq \frac{h}{h_1} \equiv F \qquad (11.2a)$$

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* Reference 4, Figure 2.

** For the values of h/λ_o and h_1/λ_o with which we are concerned (between 0.09 and 0.18), this introduces an error not exceeding 6 per cent. In view of the other uncertainties, Eq. (11.2a) is an admissible approximation. For other values of the depth-to-wave-length ratio, Eq. (11.2) should be used.

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Eq. (11.2a) is consistent with the indication of our small-scale data that, for very shallow water, γF is approximately proportional to the depth.

It will be noticed that when the approximation (11.2a) is valid, F is independent of λ_o. However, to make sure that the depth condition $h < 0.2\,\lambda_o$ is satisfied, we need to know λ_o, the wave length in deep water, at least roughly. (This wave length differs, in general, from that in shallow water; hence we cannot use here the wave lengths in Table 3 or 4.) From a curve in one of the Project Seal reports, $\lambda_o = 12$ ft. is judged to be a reasonable value* for a burst of 8 oz. Thus our experimental depths of 1 and 2 ft. of water satisfy the condition $h < 0.2\,\lambda_o$.

Equations (11.1) to (11.2a) are non-dimensional. If we now assume that the foregoing considerations also apply to large-scale bursts, such as those in Figure 26, we can easily deduce the factor F for the same explosions in shallower water. The depths in Figure 26 are scaled up from those in our pond experiments. Hence these, as well as lesser depths, should also meet the requirement $h < 0.2\,\lambda_o$.

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*This figure was obtained as follows: In Fig. 17, Reference 7 the wave length is about 24 ft. for a surface burst of 4.25 lbs. Scaling this down to 0.5 lb. by the cube root of the charge weight ratio, we obtain, approximately, $\lambda_o = 12$ ft.

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Table 8. Estimated Values of $\overline{yr}$ for a Surface Explosion. in Water 150 ft. Deep.

W (kilotons)	h_1 (ft)	$\overline{y_1 r_1}$ (ft^2)	$F^* = \frac{h}{h_1}$	$\overline{yr}$ (ft^2)
10	180	1.13×10^5	0.85	0.96×10^5
20	210	1.61 x "	0.70	1.13 x "
50	265	2.53 x "	0.55	1.39 x "

*To the nearest 0.05. Greater precision is not justified.

Table 8 lists values of F and $\overline{yr}$ for a depth h = 150 ft. (which is understood to be fairly typical of the Bikini lagoon), for three bomb efficiencies expressed in TNT equivalents. The first three columns are identical with those in the first half of Table 6. Figure 28 was obtained from Figure 26 by multiplying the latter's ordinates by F (i.e., by 0.85, 0.70, and 0.55 respectively, for the 10, 20, and 50 kiloton curves). Figure 26 was chosen for this purpose, rather than Figure 27, because considerably less extrapolation is required from the depths in the former than from those in the latter. The amplitudes in Figure 28, like the others in this report, are crest-to-trough heights. It should be emphasized that these estimates refer to a bomb detonated at the <u>surface.</u>

We next make a rough estimate of the amplitudes to be expected in deep water. For 8 oz. in 2 ft. of water, $h/\lambda_o = 1/6$, if we again take $\lambda_o = 12$ ft. Substituting this into Equ. (11.1) we have

$$\frac{yr}{y_o r_o} = 2.4\ (1/6) - 0.01 = 0.39$$

or $y_o r_o = 2.6$ yr. Assuming again that these considerations are applicable to large-scale bursts, this means that, at a given distance in deep water, the amplitudes of the waves from a nuclear-bomb explosion will be approximately 2.6 times as great as those in Figure 27. It would be better merely to say "two or three times as great," in view of the many uncertainties in this analysis.*

Before leaving this subject, it is necessary to specify what is meant here by "deep" water. In the same Seal graph on which Eq. (11.1) was based, $yr = y_o r_o$ when $h \geq 0.8\ \lambda_o$. Thus, scaling up $\lambda_o = 12$ ft. by the appropriate values of m in Table 5,

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*Apart from the extrapolations involved, it should be noted that λ_o, which is not accurately known, enters explicitly in determining the value 2.6. This is unlike the situation in very shallow water, where, as we have seen, F is independent of λ_o.

and multiplying by 0.8, we arrive at the following definitions of "deep" water:

For 10 kilotons, a depth ≥ 1450 ft.

For 20 kilotons, a depth ≥ 1700 ft.

For 50 kilotons, a depth ≥ 2150 ft.

12. Discussion.

Surface vs. Underwater Bursts.—The principal result of these experiments is that in shallow water, charges detonated at the surface produce higher waves than submerged charges. The New Zealand Seal Project found this to be true also for deep water: surface bursts generate higher waves* than those produced even by a charge at Penney's "optimum depth."[6)]

Seal also observed that as an 8 oz. charge is submerged from 0 to a few inches in deep water, the wave amplitudes decrease. British investigators[3)] found that as a charge is submerged from about 0.25 R_m (where R_m is the maximum bubble radius) to 0.9 R_m, the amplitudes first increase, then reach a maximum, and finally decrease, the peak occurring at about 2/3 R_m. Actually there is no contradiction between these two sets of results. The range of depths in the British experiments did not include depths very close to the surface, nor—and this is most important—did they try any bursts at the surface. This

*Reference 4, Figure 1.

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explains why they concluded that 2/3 R_m is the optimum depth for wave production.

Combining the results of these experiments with our own, and with those of the Seal Project, one arrives at the following conclusions: If, in a given situation, the principal aim of an explosion in water is to produce gravity waves of maximum amplitude, the place at which the charge should be fired is <u>at the surface</u>.* As the submergence of the charge is increased to about 0.1 R_m, the amplitudes decrease rapidly. At greater depths, the amplitudes increase slowly, reaching a secondary maximum at about 2/3 R_m. This maximum is, however, distinctly lower than that at the surface.

<u>Estimates of Nuclear-Bomb Effects</u>.—The predictions in Sections 10 and 11 should be regarded as order-of-magnitude estimates only. To be sure, these estimates were not obtained by indiscriminate extrapolation; we did not apply the small-charge scaling laws to a comparison of large bursts with small ones. Nevertheless, the scaling laws which we did use to compute nuclear-bomb effects, are based on an hypothesis which lacks experimental confirmation in the domain of large-scale explosions. This hypothesis of a mechanism

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*or slightly below it. Project Seal concludes[4] that, for optimum wave production, the center of gravity of the charge should be submerged to a depth of 0.05 $W^{1/3}$ ft. (where W is in pounds). For a half-pound charge, this is a depth of 0.5 inch. For a 20,000-ton bomb, this corresponds to a depth of 17 ft. The wave heights will be much the same for any location of the charge between this depth and the surface.

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of wave production by surface explosions is consistent with the results of small-scale experiments. It cannot, however, be considered established until it has been tested for surface bursts of the order of 100 tons or more.

Efficiency of Energy Conversion.—According to Project Seal*, the efficiency of conversion of explosive into wave energy increases steadily from small charges up to those of several thousand pounds, where it levels off. This conclusion is based on the supposition that a $W^{\frac{1}{4}}$ scaling law subsists throughout this range of charge weights. The actual amplitudes were observed to increase faster than predicted by the $W^{\frac{1}{4}}$ law, and this unexpected rate of increase was attributed to an enhanced conversion efficiency for larger charges.

However, an alternative explanation of the discrepancy is possible: that the fourth-root law is not applicable over the entire range of charges in question. If, instead of this law, a cube-root law is adopted for small charges, and a law conforming to Eq. (9.2) is used for charges of intermediate weight, most of the disproportionate increase in amplitude is accounted for, and most of the supposed increase in conversion efficiency disappears.

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*Reference 4; see especially Figure 8.

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It should be added, in this connection, that in their earlier papers, Seal tentatively reported a $W^{1/3}$ law for charges up to 300 lbs. Later,[4,7] on theoretical grounds, they adopted a $W^{1/4}$ law, but asserted that so far as their experimental data went, "the cube-root law...is a closer fit than the fourth-root law."[7]

Addendum

Dr. K. Fuchs has called the writer's attention to the following difference between the surface explosion of a nuclear bomb and that of a corresponding charge of TNT. In the latter, the distance from the source at which the pressure becomes acoustic is not very large relative to the dimensions of the H.E. Therefore, the energy which was able to get into the water--by virtue of the relative densities of H.E. and water--can stay there long enough to produce a large cavity. In the nuclear-bomb explosion, on the other hand, a considerable amount of energy may escape from the water to the air before acoustic pressure is reached. This would reduce the cavity size, and therefore the height of the waves.

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A P P E N D I X

"Anomalous" Results

Some of the results obtained in these experiments appear rather curious at first sight. These can, however, be explained by considering the behavior of the gas globe produced by an underwater explosion, together with certain generalizations about the production of waves by explosions. We shall first list the apparently anomalous results, using, for convenience, the following notation (*cf*. Table 9): S-, B-, and M-waves will refer to surface gravity waves generated by firing charges at the surface, bottom, and midway between surface and bottom, respectively. The subscripts 1 and 2 will denote the total depth of water in feet. Thus S_2- waves will refer to waves produced by firing a charge at the surface of water 2 ft. deep. A ratio such as S_2/B_2 will denote the average ratio of the corresponding wave amplitudes from a charge at the surface of 2 ft. of water and from a charge fired at the bottom in the same depth of water. (Paragraphs 2 to 5 below are based on Table 9.)

1. Perhaps the most surprising set of curves is the one in Figure 8. There it is seen that 2 oz. of H.E. generates B_2- waves larger than those from 4 oz., and approximately as large as those from 8 oz. Moreover, even the 1 oz. of explosive yields B_2- waves which are not much smaller than those from 4 or 8 oz.

[redacted]

TABLE 9

Average Ratios of Amplitudes for Various Conditions of Charge Immersion*

Weight of Charge (oz.)	S_1/B_1	S_2/B_2	M_2/B_2
1	2.7 ± 0.5†	1.5 ± 0.2	0.5 ± 0.1
2	3.3 ± 1.0	1.2 ± 0.2	0.7 ± 0.1
4	1.6 ± 0.3	2.4 ± 0.5	1.0 ± 0.2
8	1.1 ± 0.1	3.7 ± 0.6	2.3 ± 0.3
	S_2/S_1	B_2/B_1	
1	1.1 ± 0.2	2.1 ± 0.2	
2	1.0 ± 0.1	2.9 ± 0.4	
4	1.8 ± 0.3	1.2 ± 0.2	
8	2.1 ± 0.3	0.5 ± 0.1	

*S, B, and M refer to amplitudes of waves from charges fired at the surface, bottom, and midway between surface and bottom, respectively. Subscripts 1 and 2 denote the total depth of water in feet.

†Mean deviations from the ratios are given throughout.

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2. The ratio S_2/B_2 (cf. Table 9) is greater for 4- or 8-oz. charges than for 1- or 2-oz. charges. Exactly the converse is true of the ratio S_1/B_1: It is greater for 1 or 2 oz. than for 4 or 8 oz.

3. A comparison of M_2- and B_2-waves shows that for 1 and 2 oz. the former are smaller than the latter; for 4 oz. the two are equal in height; whereas for 8 oz., the M_2-waves are more than twice as high as the B_2-waves.

4. S_2- and S_1-waves are about equal in magnitude for 1 or 2 oz., but the former are about twice as high as the latter for 4 or 8 oz.

5. For charges fired at the bottom, 1 or 2 oz. yield B_2-waves which are two or three times as high as the B_1-waves. For 4 oz. the two types of waves are nearly alike, whereas 8 oz. produces B_2-waves which are actually smaller than the B_1-waves.

At first sight, many of these results seem paradoxical. In an effort to explain them, we shall (a) review some well-established facts about underwater explosions; (b) state several generalizations (citing supporting evidence) about the generation of waves by explosions; and (c) list the controlling factors and secondary factors which govern the production of waves under each set of conditions tried in our experiments.

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The phenomena associated with an underwater explosion have been extensively studied, and their main features are fairly well understood.[8-11] When a charge of high explosive is fired underwater, the combustion products form an expanding bubble of gas. The expansion continues until the pressure inside the bubble falls below the surrounding hydrostatic pressure (an inertial effect), and then the excess pressure outside the gas globe causes it to contract. If the charge is immersed deeply enough, the sequence of expansion and contraction may be repeated several times. Thus there occurs a series of pulsations, each accompanied by a shock wave and by a mass motion of the surrounding water. Meanwhile the gas globe tends to rise toward the surface because of its buoyancy. If the charge is initially located fairly close to the surface, the expanding bubble breaks through, venting some of its gases and inhibiting the process of pulsation.

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8) "Experiments on the Pressure Wave thrown out by Submarine Explosions," by H. W. Hilliar, British Admiralty Research Experiment 142/19, 1919.

9) "The Pressure-Time Curve for Underwater Explosions," by W. G. Penney, Ministry of Home Security, Civil Defense Research Committee, England, R.C. 142, November 1940.

10) "Theory of the Pulsations of the Gas Bubble Produced by an Underwater Explosion", by Conyers Herring, NDRC CONFIDENTIAL Report C4-sr20-010, Columbia University, Division of National Defense Research, New London, Connecticut, October, 1941.

11) "Report on Underwater Explosions," by E. H. Kennard, David Taylor Model Basin CONFIDENTIAL Report 480, October 1941.

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[redacted]

The maximum radius R_m of the gas globe has been found by Ramsauer[12] to obey approximately the following relation:

$$R_m = C\left(\frac{W}{P}\right)^{1/3}$$

where C is a constant depending upon the type of explosive and the fraction of energy which goes into the pulsating motion,

W is the mass of explosive,

P is the <u>total</u> pressure (including atmospheric) at the charge depth.

If R_m is expressed in ft., W in pounds, and P in equivalent feet of water, and if we take the heat of combustion of Composition C to be 1300 cal./gm., $C = 14.2$. At Los Alamos, atmospheric pressure is equivalent to 25 feet of water. If we compute R_m for a depth of 1 ft., then P 26 ft. of water, and R_m is given in Table 10 for the charge weights used in our experiments:

Table 10

Charge Weight (oz.)	R_m (ft.)
1	1.9
2	2.4
4	3.0
8	3.8

12) C. Ramsauer, Ann. D. Physik, <u>72</u>, 265 (1923)

[redacted]

 [redacted]

For a charge depth of 0 ft. or 2 ft., R_m differs by less than 2 per cent from these values.

In computing R_m for a charge depth of 1 foot, we have ignored the requirement that the water be deep. Actually, the values of R_m in Table 10 would be correct for charges fired at a depth, say, 15 feet below the surface, and if the total pressure (including atmospheric) at that depth were equivalent to 26 feet of water. In our experiments the second requirement was satisfied, but not the first. Since the depth of immersion was only 1 or 2 ft., the gases could not expand to a spherical globe with radius R_m. It would seem that before attaining this radius, the gas bubble would reach the surface and be vented.

The behavior of the gas globe in shallow water is, however, by no means as simple as this. It has been shown[13-16] that in the

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13) These phenomena have been explained theoretically by C. Herring, G.I. Taylor, and others, and have been discussed in detail by E. H. Kennard in Report R-182 of the David Taylor Model Basin, "Migration of Underwater Gas Globes due to Gravity and Neighboring Surfaces," December, 1943.

14) "Underwater Explosion Phenomena" and "Small-Scale Underwater Explosions under Reduced Atmospheric Pressure," David Taylor Model Basin motion pictures.

15) "Small-Scale Underwater Explosions Under Reduced Atmospheric Pressure," by Lt. D.C.Campbell, USNR, and C. W. Wyckoff, The David W. Taylor Model Basin CONFIDENTIAL Report 520, November, 1943.

16) "Motions of a Pulsating Gas Globe Under Water--A Photographic Study," by Lt. D.C.Campbell, USNR, The David W. Taylor Model Basin CONFIDENTIAL Report 512, May, 1943.

[redacted]

[redacted]

vicinity of a rigid boundary, the gas sphere tends to move toward that boundary, whereas near the surface of the water the bubble behaves as though repelled from the surface. Moreover, if a charge is fired sufficiently close to a rigid surface, the resulting gas bubble actually tends to flatten itself against the surface (cf. Figure 1 in reference[13]). These phenomena have been directly observed by means of high-speed cinematography.[14] For small charges fired in depths of 1 or 2 ft. of water, as in our experiments, we may expect these boundary effects to be very pronounced. As we shall see, it is in fact essential to take them into account in order to explain, even qualitatively, the results described in this Appendix.

It will be useful to list several generalizations about the process of wave production by explosions in shallow water. The evidence for each will be cited below.

(a) For a given depth of water, the height of the waves generated by an explosion depends upon the diameter of the cavity which is initially created in the water.

(b) A given quantity of charge produces, in general, a larger cavity and therefore higher gravity waves when detonated at the surface than when detonated underwater. A submerged charge set

[redacted]

off near the surface behaves much like a surface charge in generating waves; the larger the charge in relation to its depth of immersion, the more nearly do its wave effects approximate those of a surface charge.

(c) A charge detonated at or near a rigid bottom produces a gas bubble which tends to flatten itself against the bottom. This delays the venting of the gases, and results in creating a cavity of larger diameter than would be produced if the charge were fired at the same depth in deep water.

(d) Provided that there is sufficient charge to produce a cavity extending to the bottom, the deeper the water, the higher will be the waves produced.

Each of these statements is supported by experimental evidence, as follows:

(a) There is a close correlation between the gravity waves produced by explosions and those produced by the collapse of cavities created by other means, as shown in Section 5 of this report.*

(b) The first statement is supported by Figures 11 to 18. The second statement is an hypothesis whose validity must be judged by its degree of success in explaining certain of the "curious" results listed above.

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*See especially Figure 22; see also Reference 2.

[redacted]

(c) See references 13 and 14.

(d) Evidence for the influence of the depth of water on the height of waves was obtained by the New Zealand "Seal" Project (<u>cf</u>. Figure 2, Reference 4).

These generalizations can be used to explain the apparently anomalous results described above. In Table 11 are listed the factors which would be expected to govern the formation of cavities and the generation of waves under the various conditions in our experiments. The values of R_m in Table 10 were taken into account in constructing the present table. The symbols s, s', F, and h denote the following:

"s" refers to the superior efficacy of surface explosions in producing waves; "s'" refers to the similar efficacy of submerged charges <u>near</u> the surface.

"F" represents the flattening of the gas bubble against a rigid surface.

"h" refers to the contribution of the depth of water to the height of the waves.

We shall now discuss in turn each of the comparisons on page 1 of Appendix:

1. The bottom of the pond at the site of the explosions was rigid, consisting of ½-inch armor plate. The gas bubble from a charge placed on the steel plate tends to be flattened against

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TABLE 11

Factors which Govern Cavity Formation and Wave Generation in Shallow Water.

Depth of Water (ft.)	Location of Charge	Weight of Charge (oz.)	Controlling Factors	Secondary Factors
2	Surface	1	s	
		2	s	
		4	s	h
		8	s	h
2	1' Below Surface	1	h	
		2	h	
		4		s', h
		8	s'	h
2	Bottom	1	F	h
		2	F, h	
		4	h	F
		8		s'
1	Surface	1	s	
		2	s	
		4	s	
		8	s	
1	Bottom	1		F
		2		F
		4		s'
		8	s'	

[redacted]

the bottom and spread horizontally. For both the 1- and 2-oz. charges in 2 feet of water this effect produces a cavity of considerable diameter in the water before the gases are vented. The 2-oz. charge appears to be close to the optimum size for "taking advantage" of the flattening and spreading effect to produce a wide, well-defined hole in water 2 ft. deep. On the other hand , the gas bubbles from the 4-oz. and 8-oz. charges, despite some flattening, break through the surface before they have had a chance to spread and thereby to increase the size of the cavity. This explains why the waves from 2-oz. charges placed at the bottom in 2 ft. of water are relatively large.

2. For the reason just cited, 4-oz. and 8-oz. charges are not very effective when fired at the bottom in 2 ft. of water; by virtue of (b), they are more effective when detonated at the surface. For 1- or 2-oz. charges in this depth, however, surface explosions are not markedly better than bottom ones, since flattening of the bubbles created at the bottom makes them effective cavity and wave producers. Hence S_2/B_2 is greater for 4- and 8-oz. charges than for 1 or 2 oz.

In water 1 ft. deep, on the other hand, surface explosions of 1 or 2 oz. are markedly better than bottom ones by (a) and because

[redacted]

the water is too shallow--even for these small charges--to permit appreciable spreading before venting. This superiority of the surface charges is smaller for 4 oz. and negligible for 8 oz. because for charges of this size, a submergence of only 1 ft. make them little different from surface charges. For these reasons S_1/B_1 is greater for 1 or 2 oz. than for 4 or 8 oz.

3. M_2-waves are smaller than B_2-waves for 1- and 2-oz. explosions because the flattening effect is more pronounced in the bottom explosions, where the charge is actually in contact with the rigid plate. For 4-oz. charges in 2 ft. of water, the bubble is rapidly vented whether the charge is at a depth of 1 ft. or 2 ft. A slight flattening effect enhances the cavity-size in the B_2 explosion; but offsetting this, in the M_2 explosion, is the proximity of the charge to the surface. Thus, for 4 oz., $M_2 \doteq B_2$. An 8-oz. charge 1 ft. deep in 2 ft. of water behaves nearly like a surface charge, whereas if it is fired at the bottom, the attraction of the bottom and greater distance from the surface result in a smaller cavity (the 8-oz. explosion produces so large a bubble that, because of venting, it cannot "take advantage" of the flattening effect in shallow water). Hence, for 8 oz., $M_2 > B_2$.

[redacted]

4. When the charges are fired at the surface, 1 or 2 oz. are apparently too small to create a cavity extending to the bottom in 2 ft. of water. Hence the cavity size is not much different from that produced in 1 ft. of water, and $S_2 \doteq S_1$. However, 4- or 8-oz. charges do produce a deeper cavity in 2 ft. of water than in 1 ft. Hence $S_2 > S_1$.

5. Comparing the B_2- with the B_1-waves from 1 or 2 oz., the flattening effect conduces more readily to the formation of a good-sized cavity in 2 ft. of water than in 1 ft; hence $B_2 > B_1$. For 4 oz. this effect in the deeper water tends to be balanced by the proximity to the surface in the 1 ft. depth, which makes the 4-oz. charge behave somewhat like a surface charge. Therefore $B_2 \doteq B_1$. Finally, for 8 oz., both depths are so shallow that the flattening effect plays a negligible role. The surface-proximity effect, however, is strong for both depths, and markedly stronger at the 1 ft. depth than at 2 ft. Thus $B_2 < B_1$ despite the greater depth in the former case.

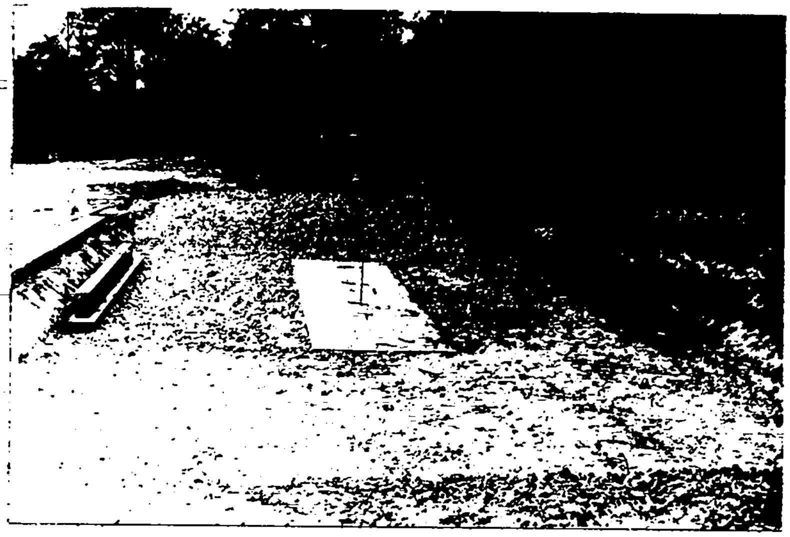
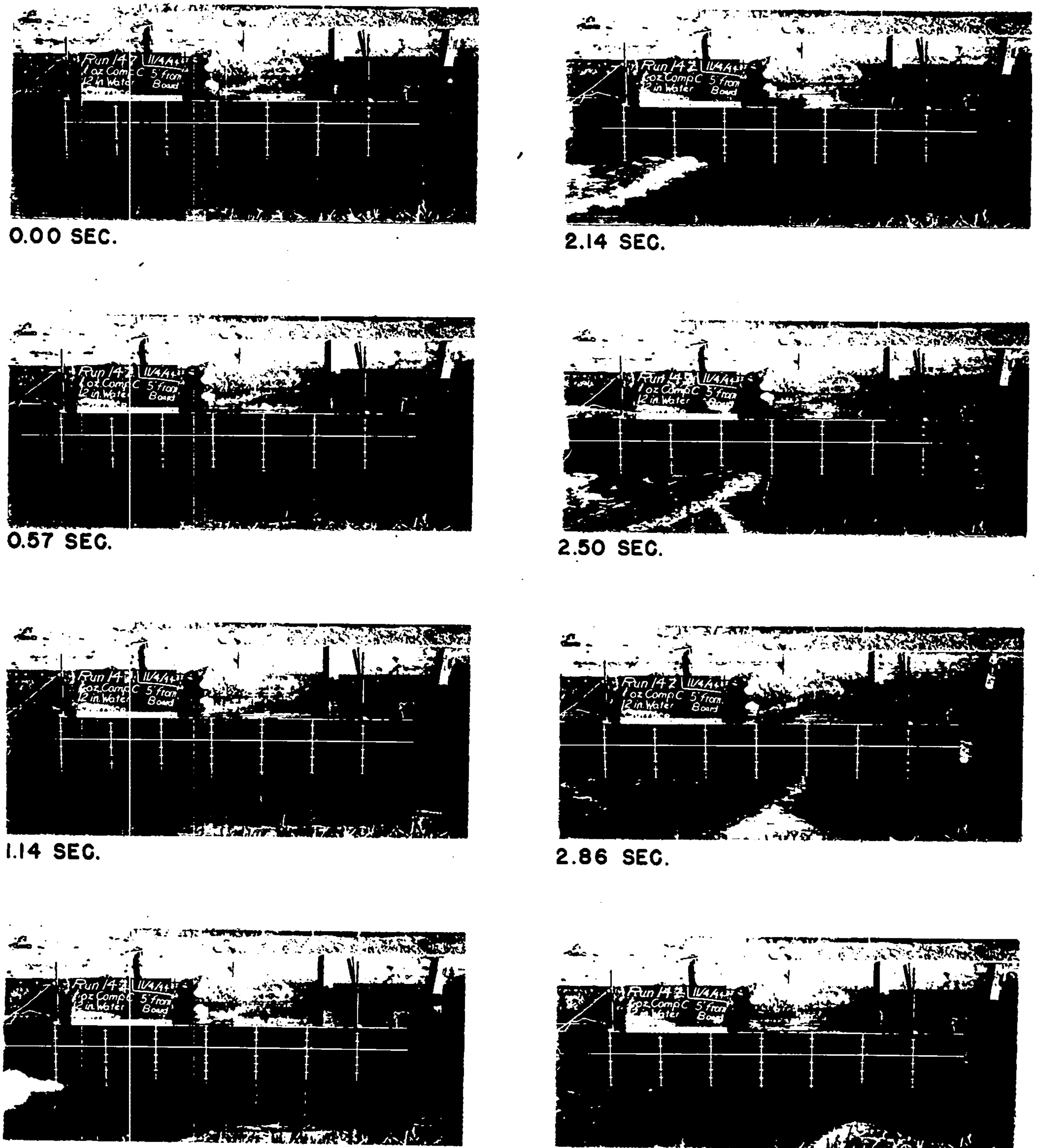


FIG. I. EMPTY POND, SHOWING STEEL PLATE ON BOTTOM.

FIG. 2. SURFACE EXPLOSION OF 8 OZ. H.E. SHOWING COORDINATE BOARD IN RELATION TO EXPLOSION

FIG. 3. WAVES GENERATED BY 1 OZ. COMP. C DETONATED AT SURFACE IN 1 FT. OF WATER



0.00 SEC.

0.57 SEC.

1.14 SEC.

1.71 SEC.

2.14 SEC.

2.50 SEC.

2.86 SEC.

3.21 SEC.

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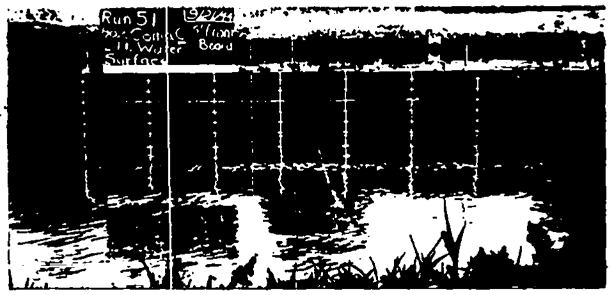
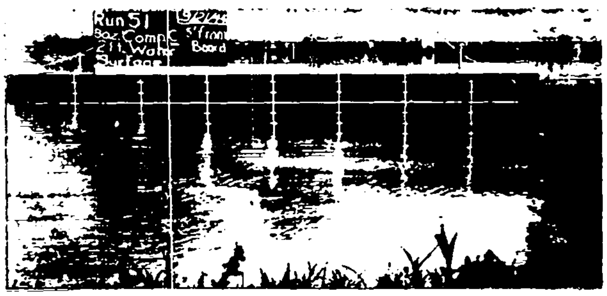
FIG. 4. WAVES GENERATED BY 8 OZ. COMP. C

DETONATED AT SURFACE IN 2 FT. OF WATER



0.00 SEC.



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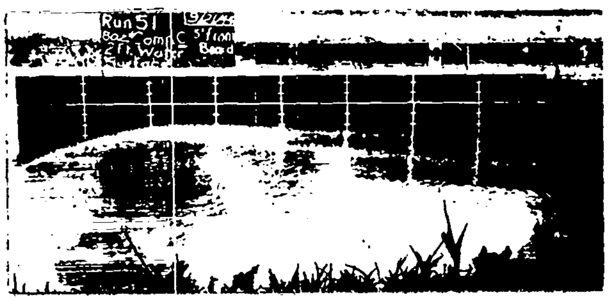

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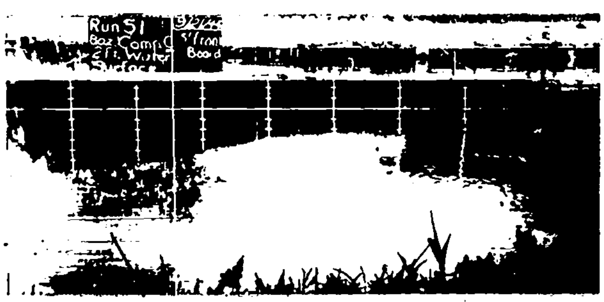
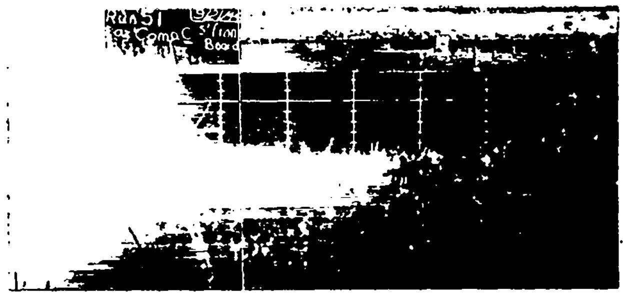
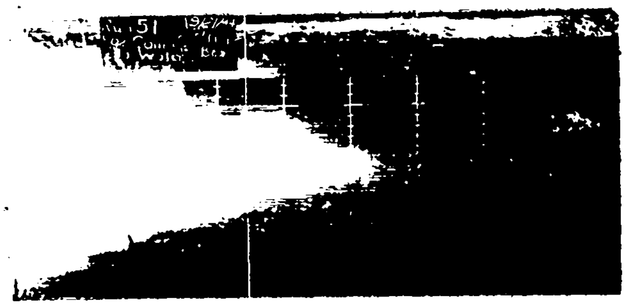


1.29SEC.

1.64 SEC.



2.00 SEC.



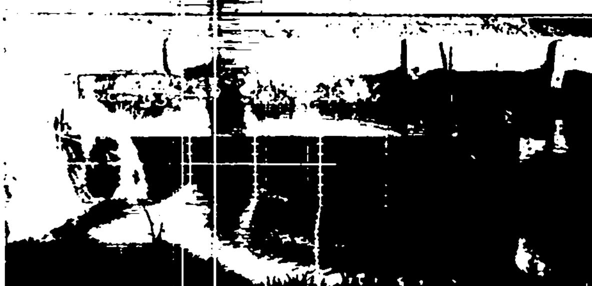
2.36 SEC.

2.86 SEC.

FIG. 5. WAVES GENERATED BY 8 OZ. COMP. C DETONATED AT SURFACE IN 1 FT. OF WATER

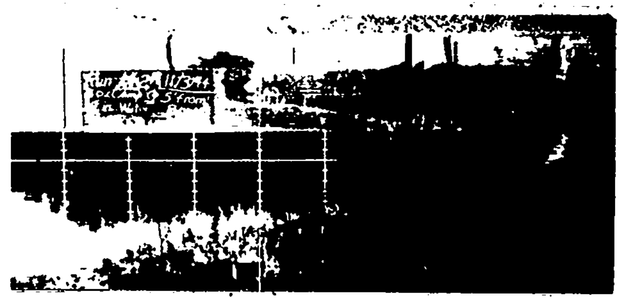
0.00 SEC.

0.36 SEC.

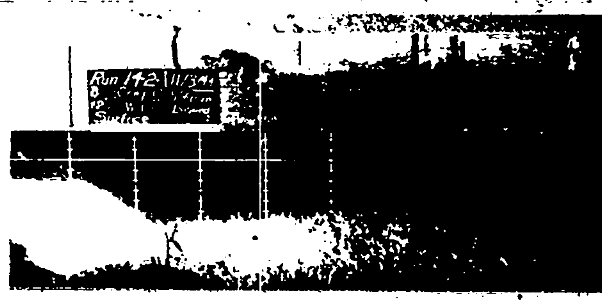


0.71 SEC.

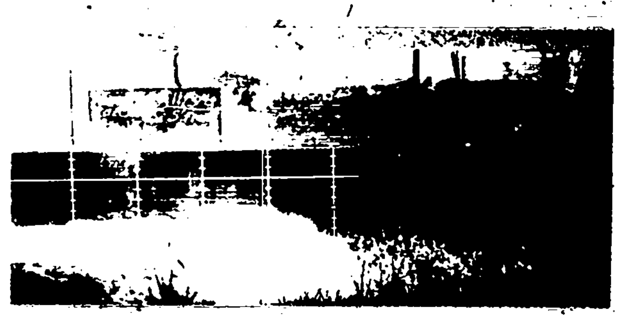
1.07 SEC.



1.43 SEC.



1.79 SEC.



2.14 SEC.

2.50 SEC.

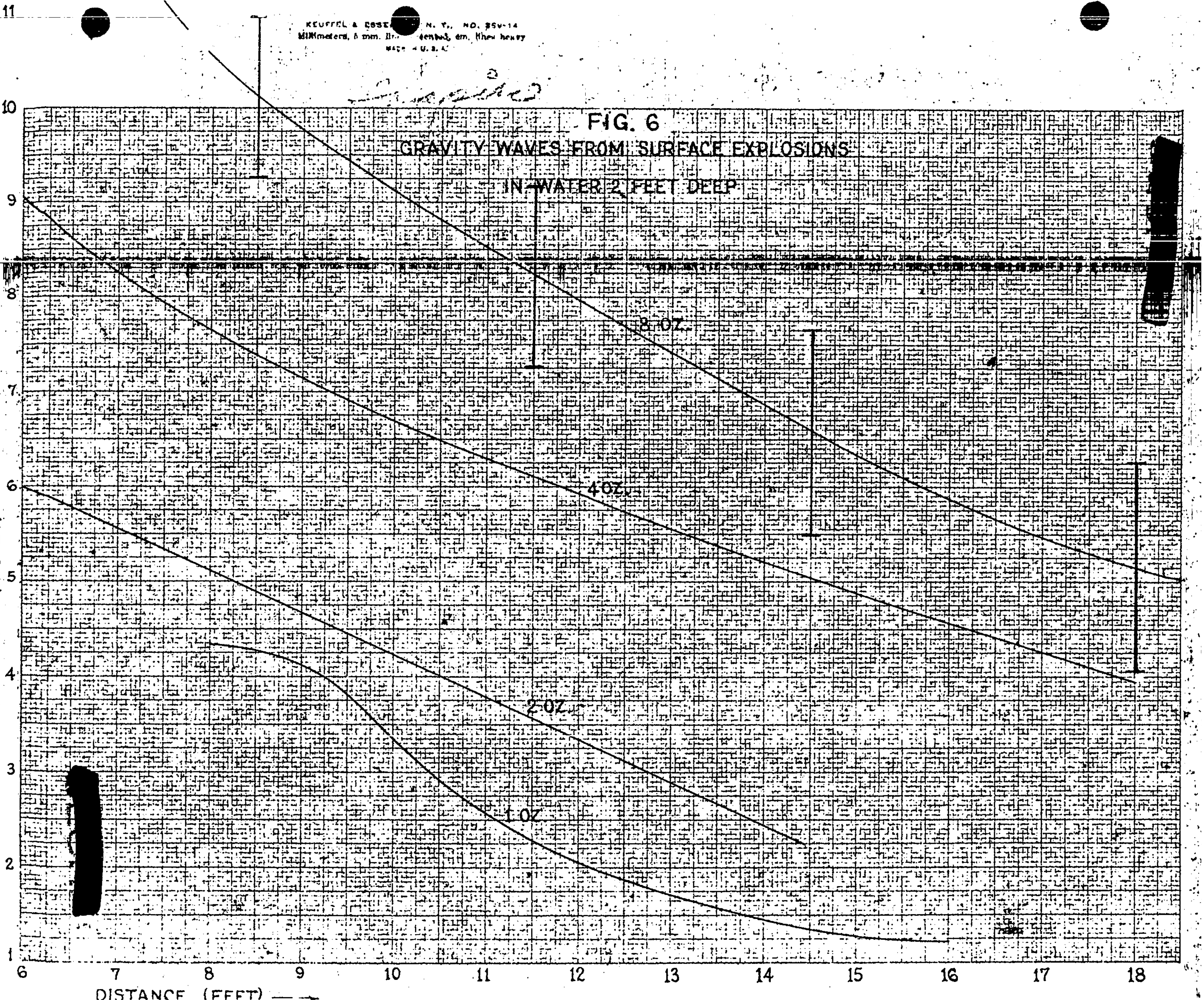


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FIG. 7

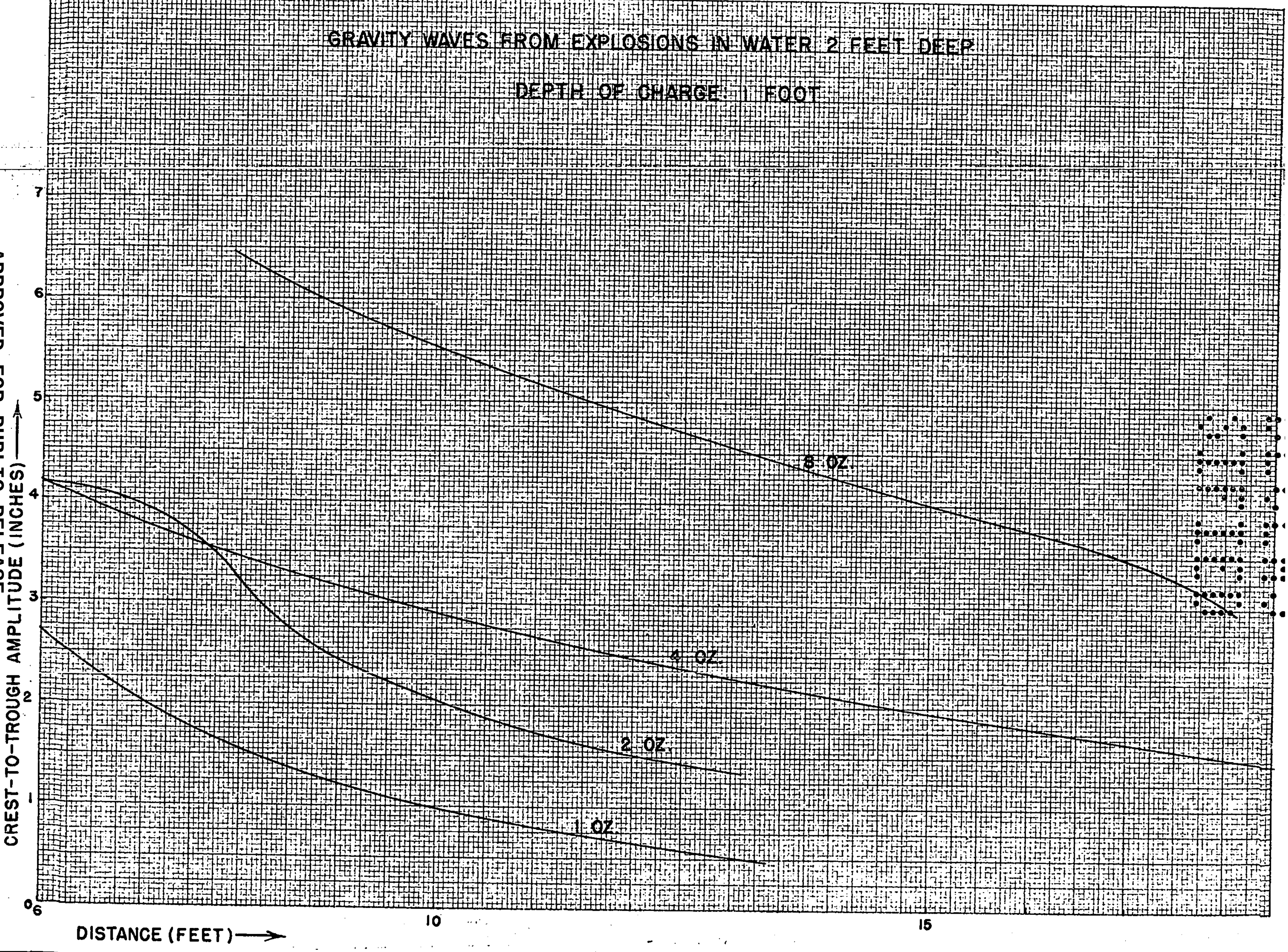
GRAVITY WAVES FROM EXPLOSIONS IN WATER 2 FEET DEEP
DEPTH OF CHARGE: 1 FOOT
8 OZ.
4 OZ.
2 OZ.
1 OZ.
CREST-TO-TROUGH AMPLITUDE (INCHES) ⟶
DISTANCE (FEET) ⟶
0
1
2
3
4
5
6
7
6
10
15

FIG 8

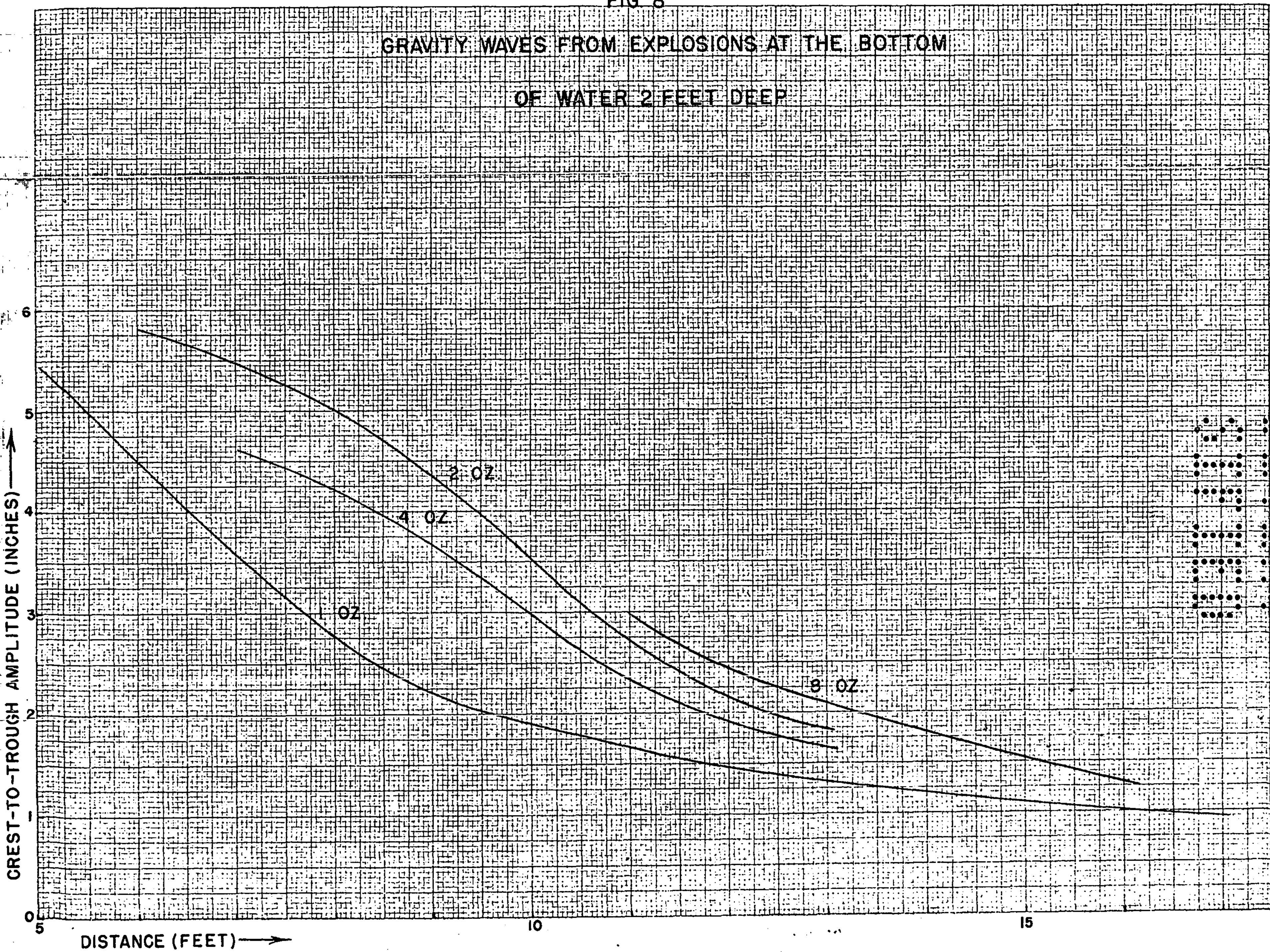

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FIG. 9

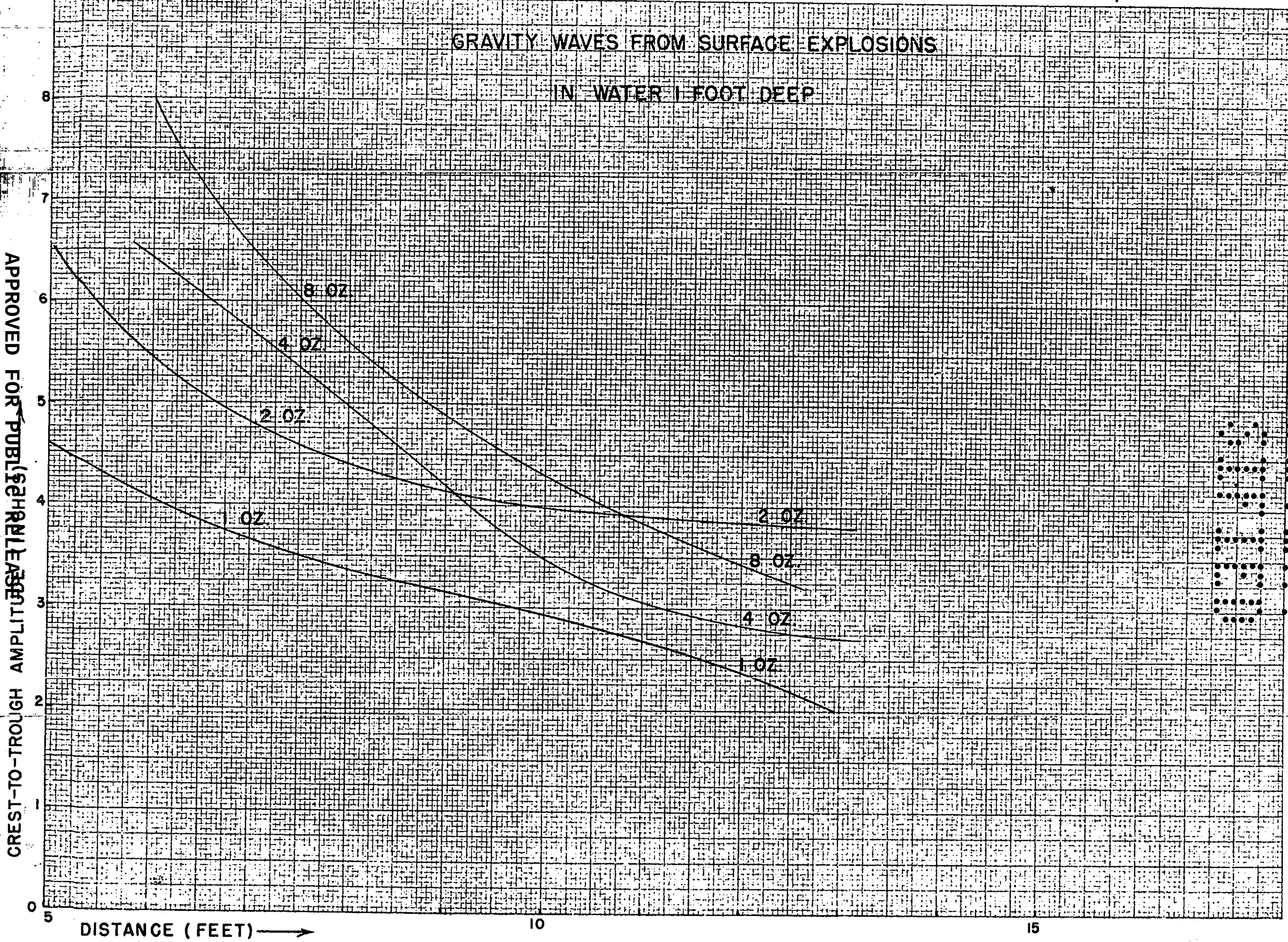
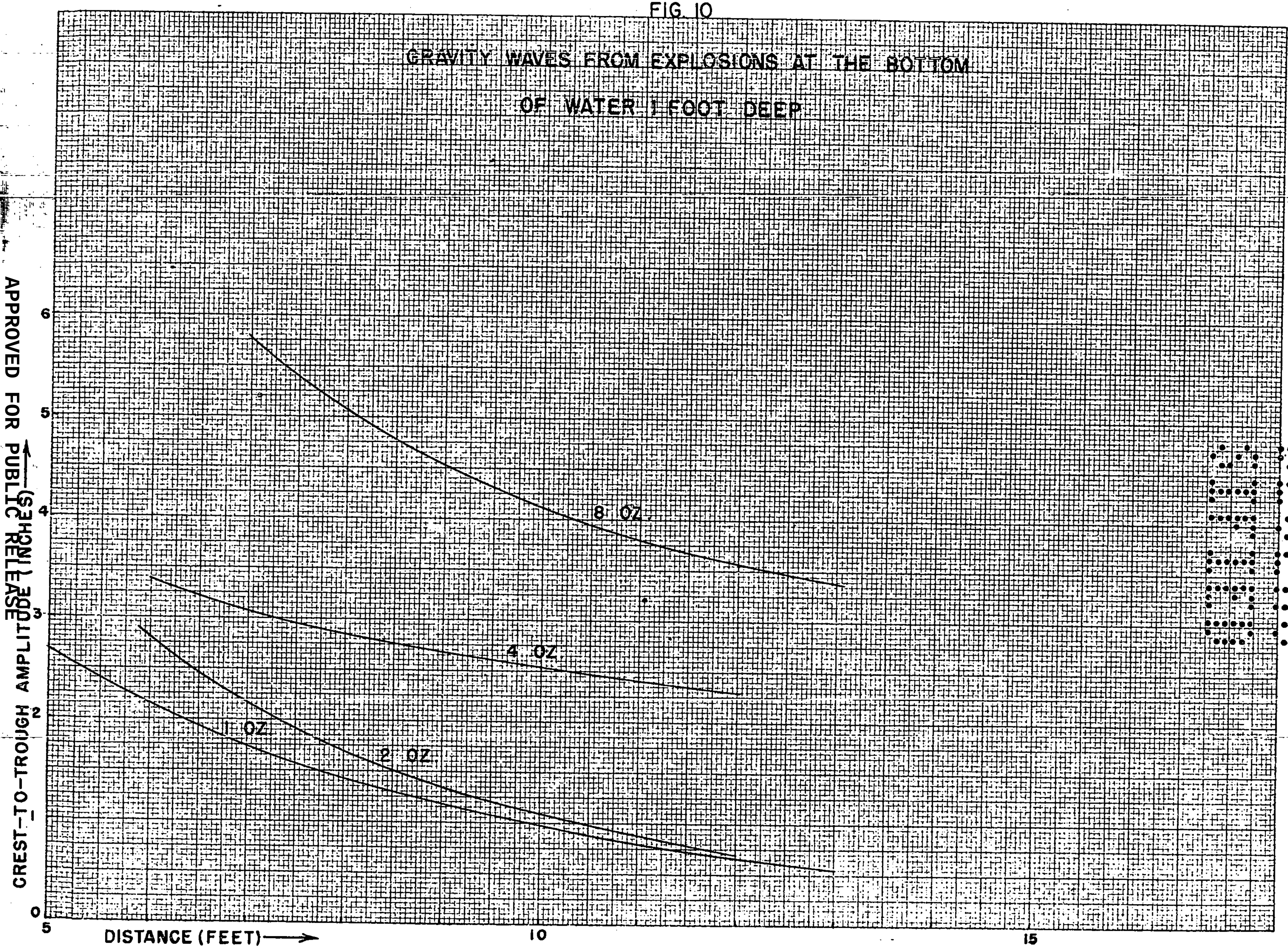

GRAVITY WAVES FROM SURFACE EXPLOSIONS
IN WATER 1 FOOT DEEP
CREST-TO-TROUGH AMPLITUDE (INCHES)
DISTANCE (FEET)
8 OZ.
4 OZ.
2 OZ.
1 OZ.
0
1
2
3
4
5
6
7
8
5
10
15

FIG 10



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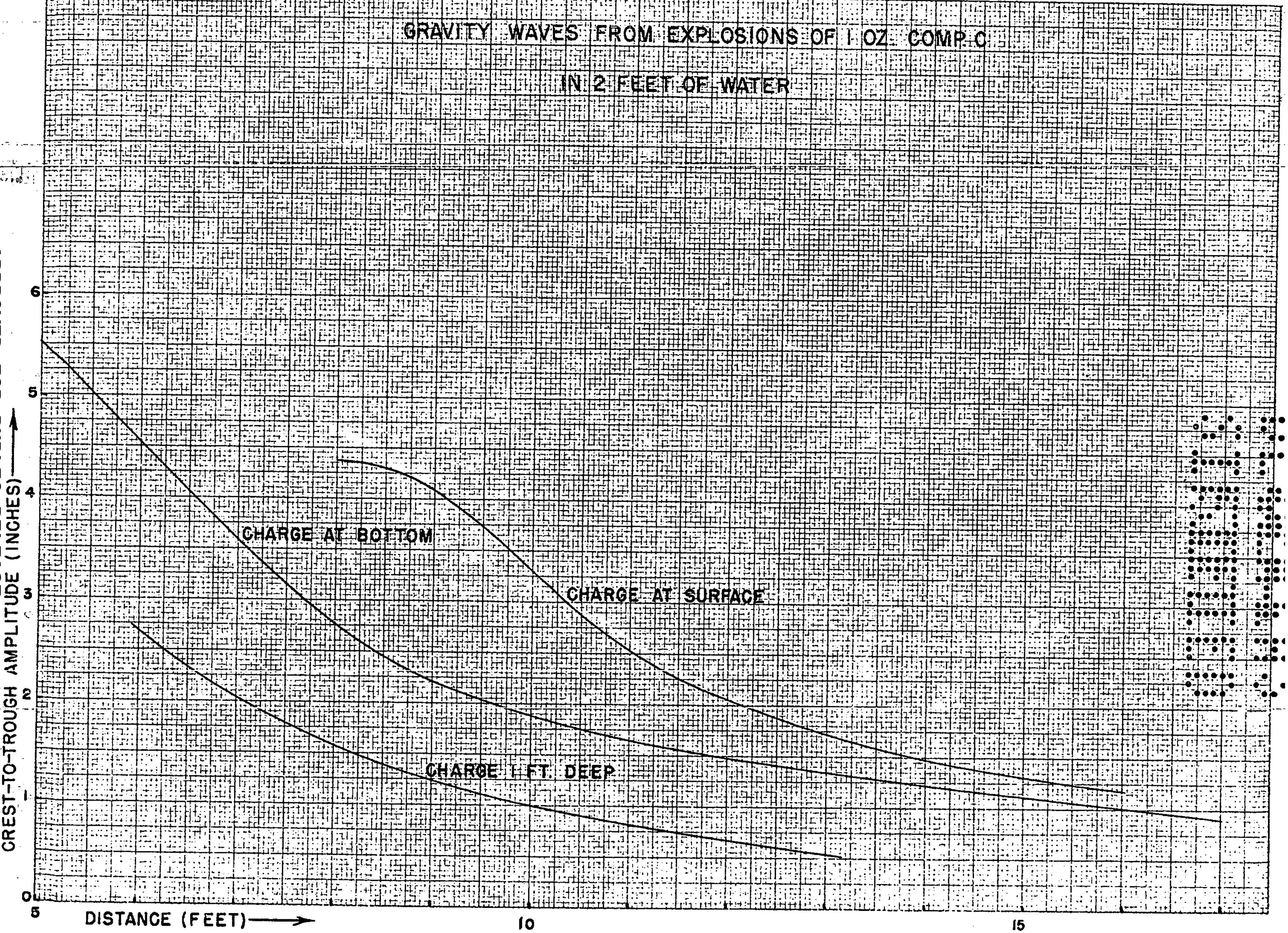
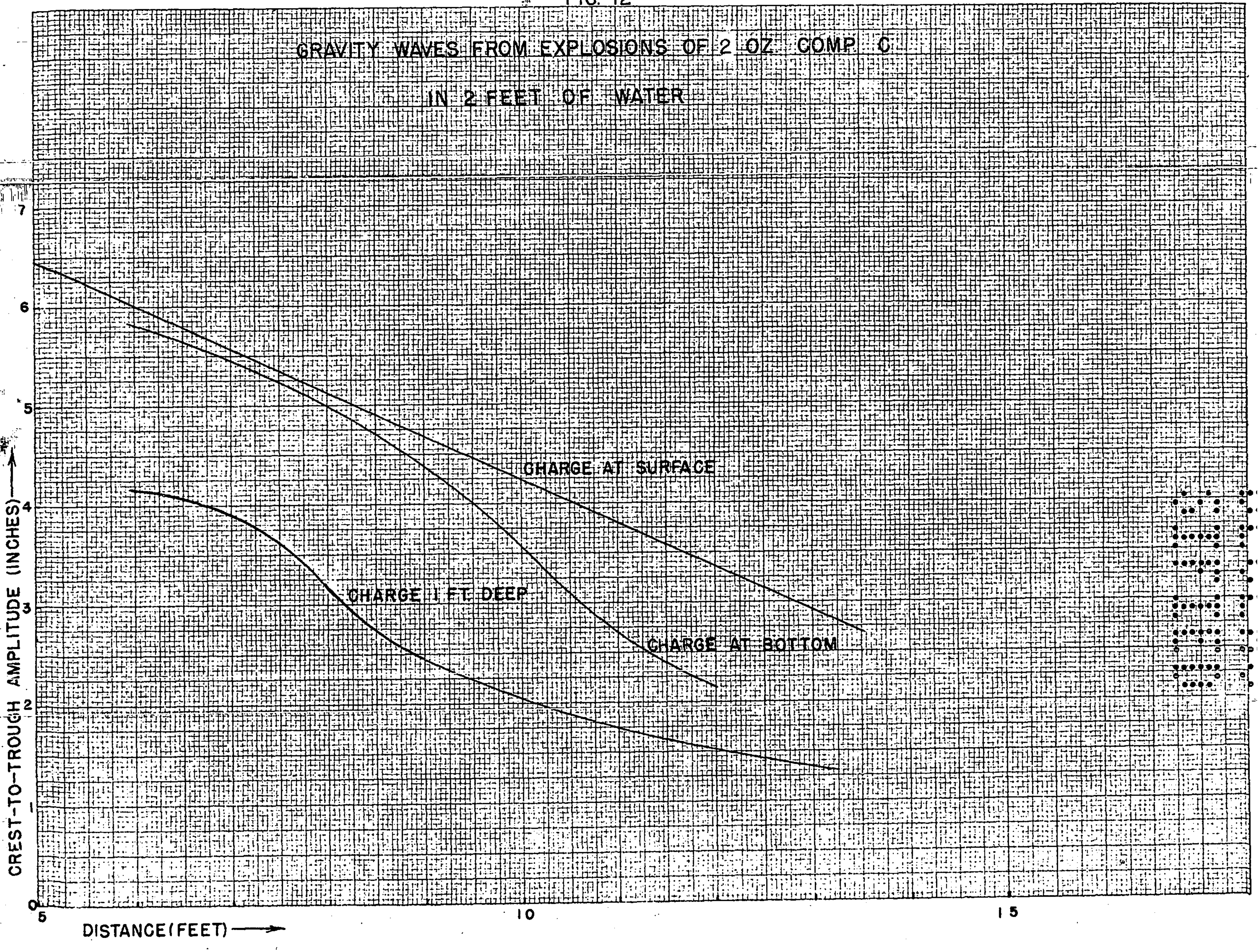
KEUFFEL & ESSER CO., N.Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U.S.A.

FIG. II



Keuffel & Esser Co., N. Y., No. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.

FIG. 12



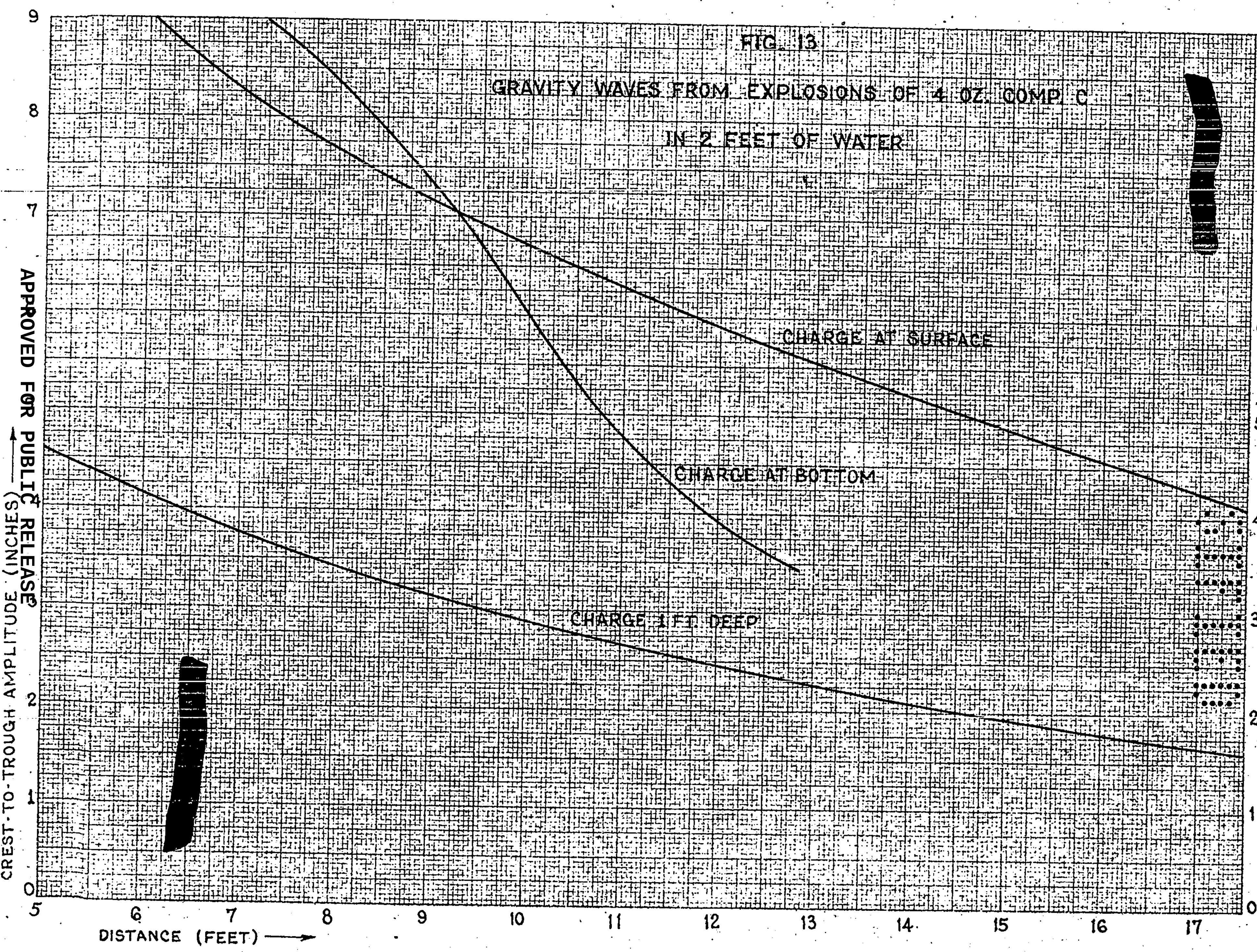


FIG. 13

GRAVITY WAVES FROM EXPLOSIONS OF 4 OZ. COMP. C IN 2 FEET OF WATER

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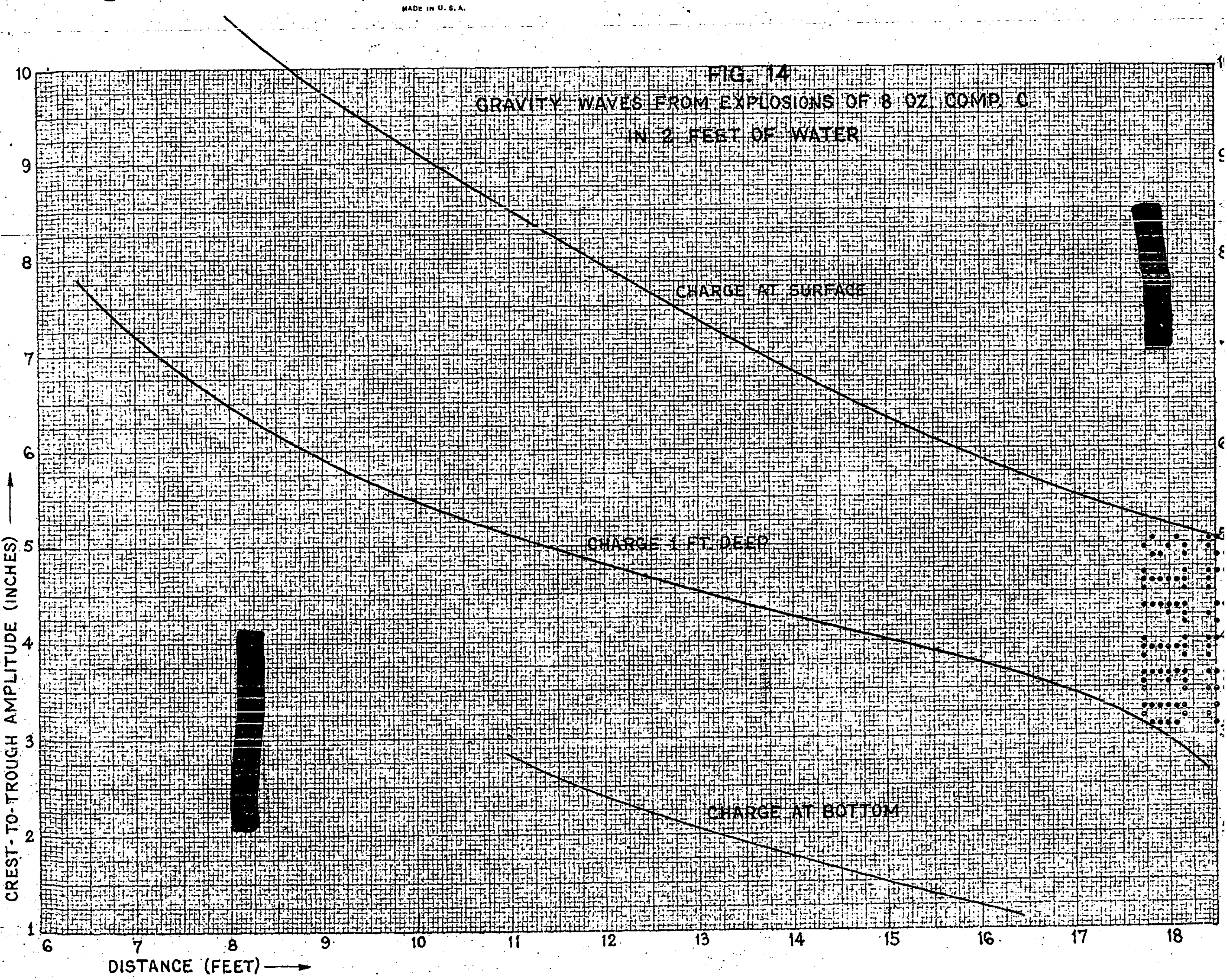

FIG. 14
GRAVITY WAVES FROM EXPLOSIONS OF 8 OZ. COMP. C
IN 2 FEET OF WATER

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FIG. 15

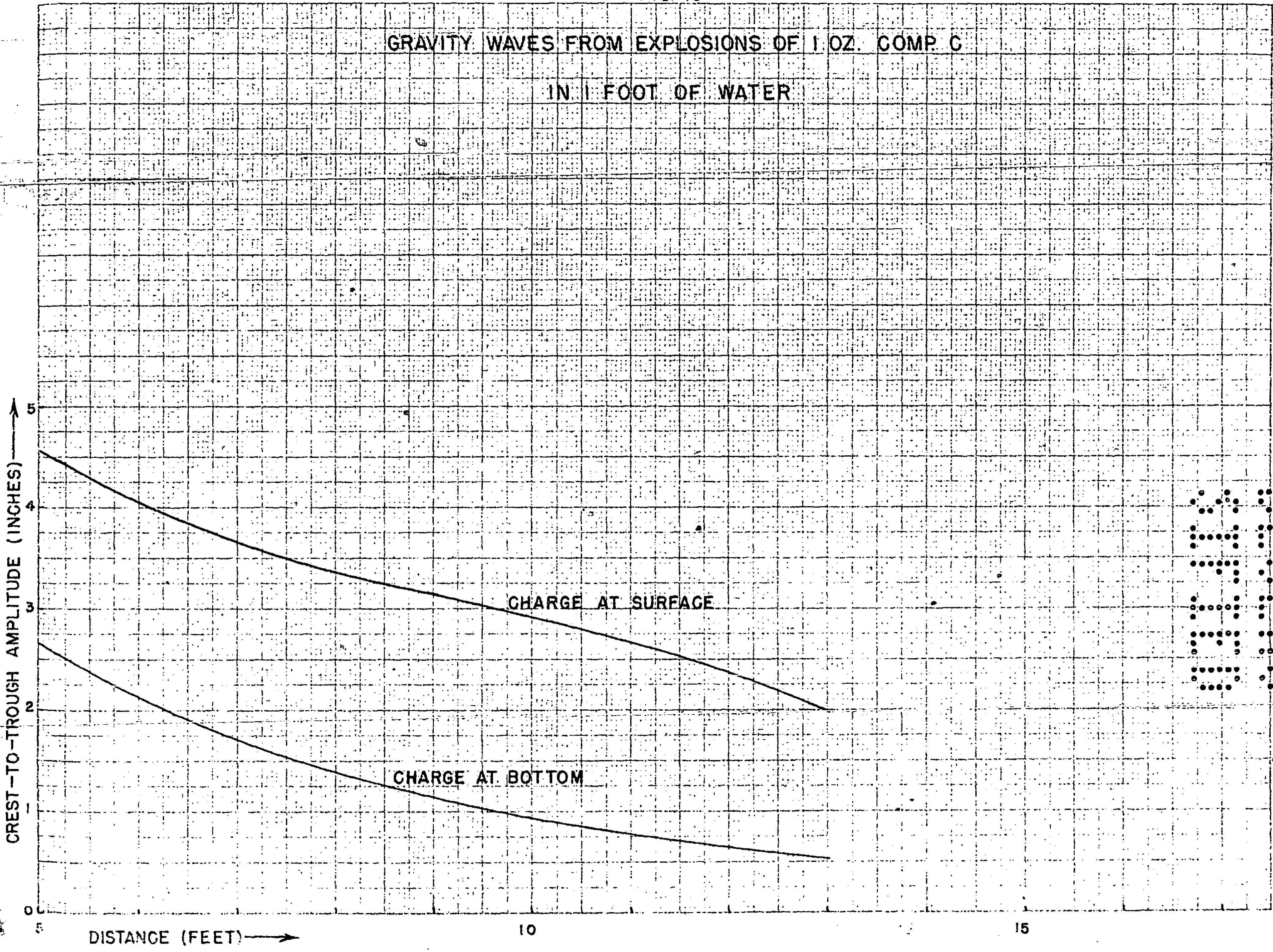

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FIG. 16

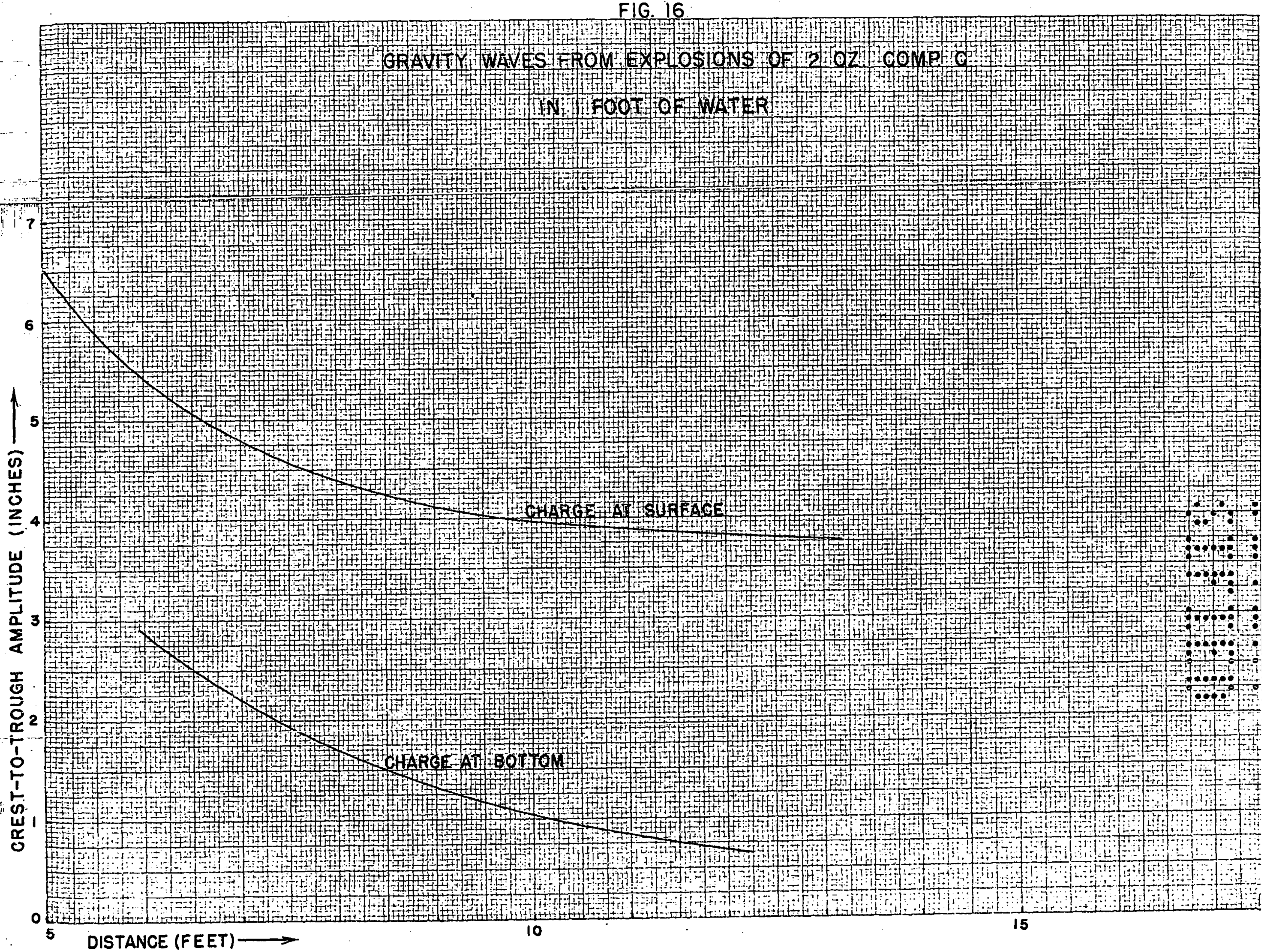

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FIG. 17

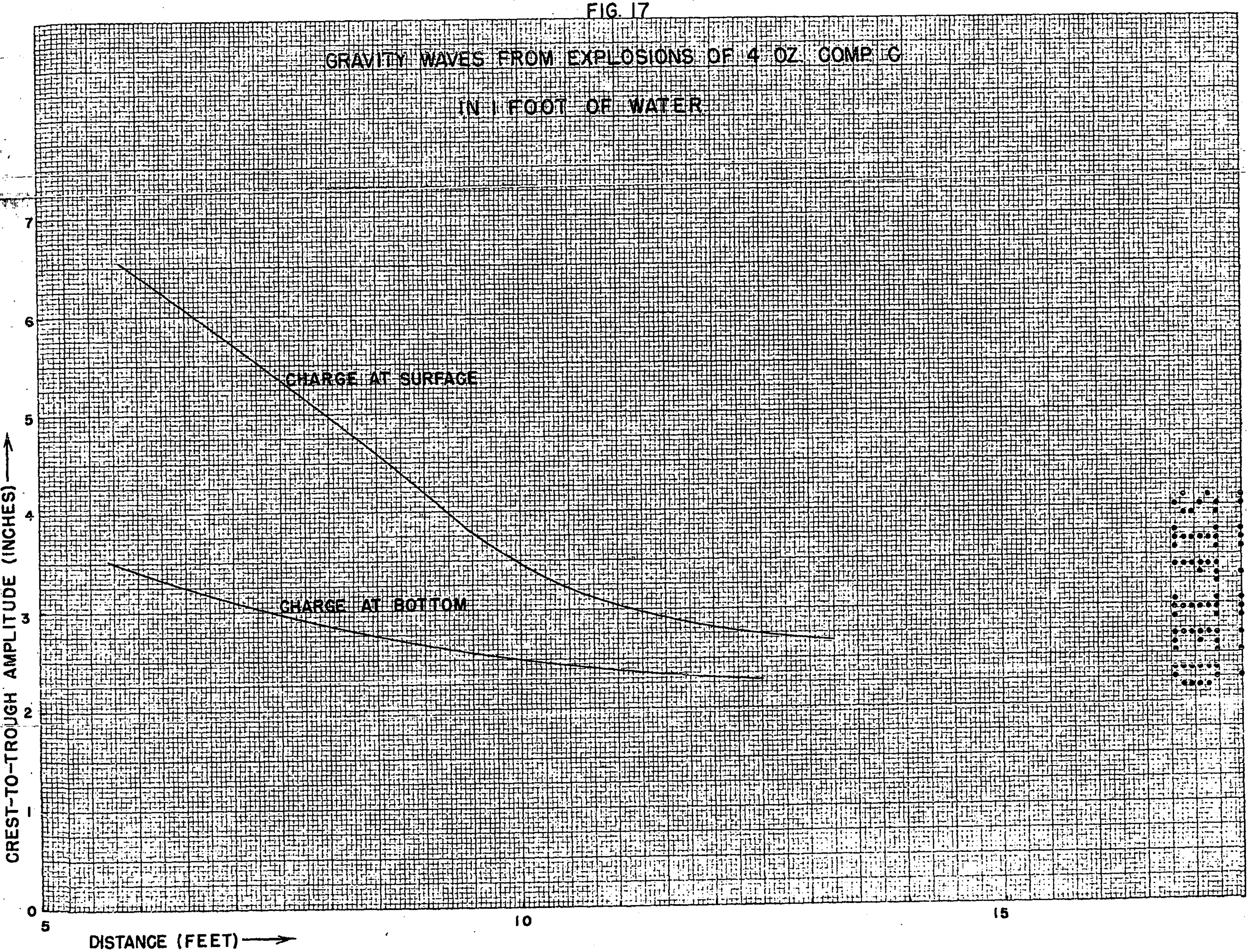
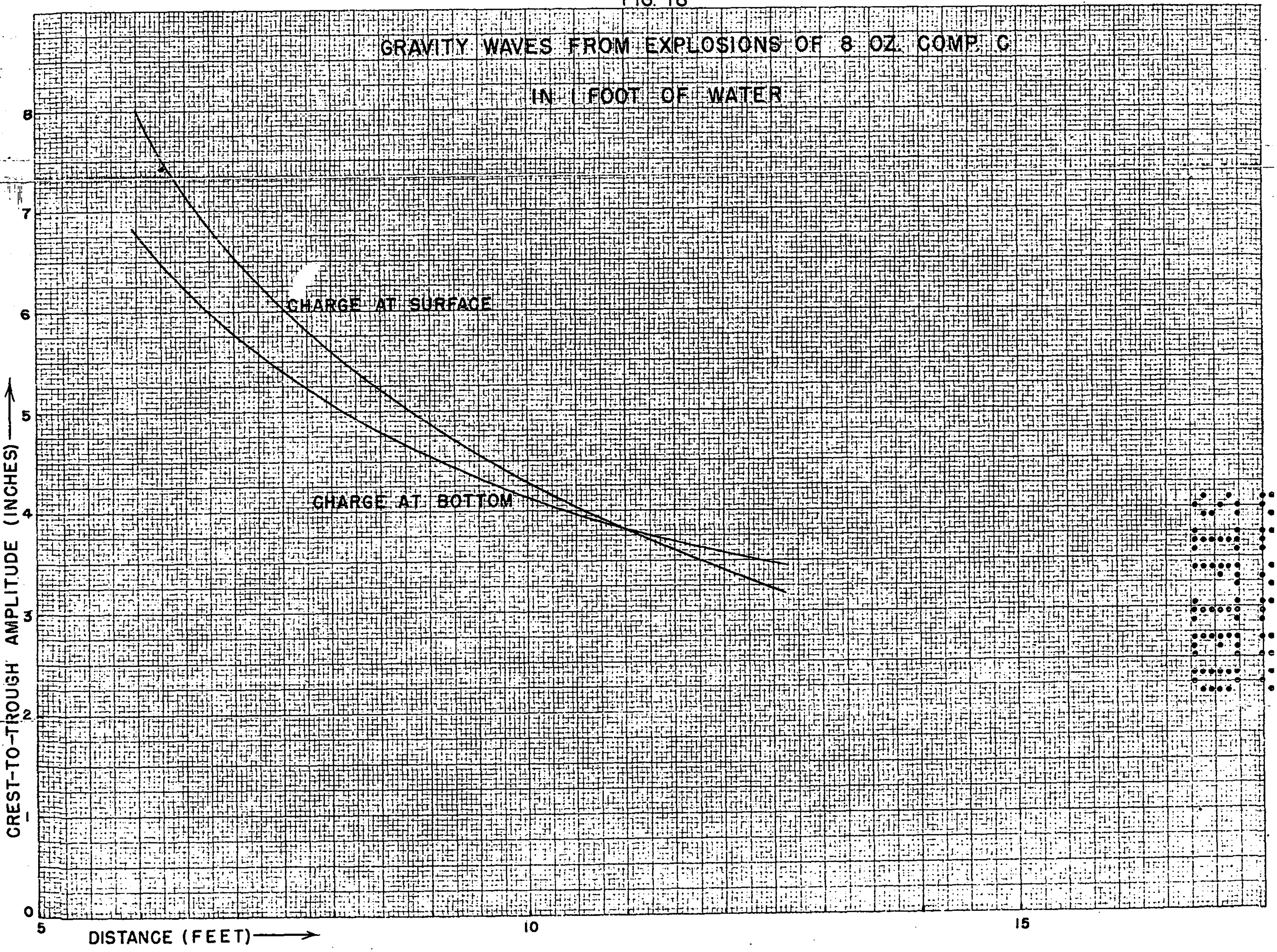

GRAVITY WAVES FROM EXPLOSIONS OF 4 OZ. COMP. C
IN 1 FOOT OF WATER
CHARGE AT SURFACE
CHARGE AT BOTTOM
CREST-TO-TROUGH AMPLITUDE (INCHES)
DISTANCE (FEET)
0
1
2
3
4
5
6
7
5
10
15

FIG. 18



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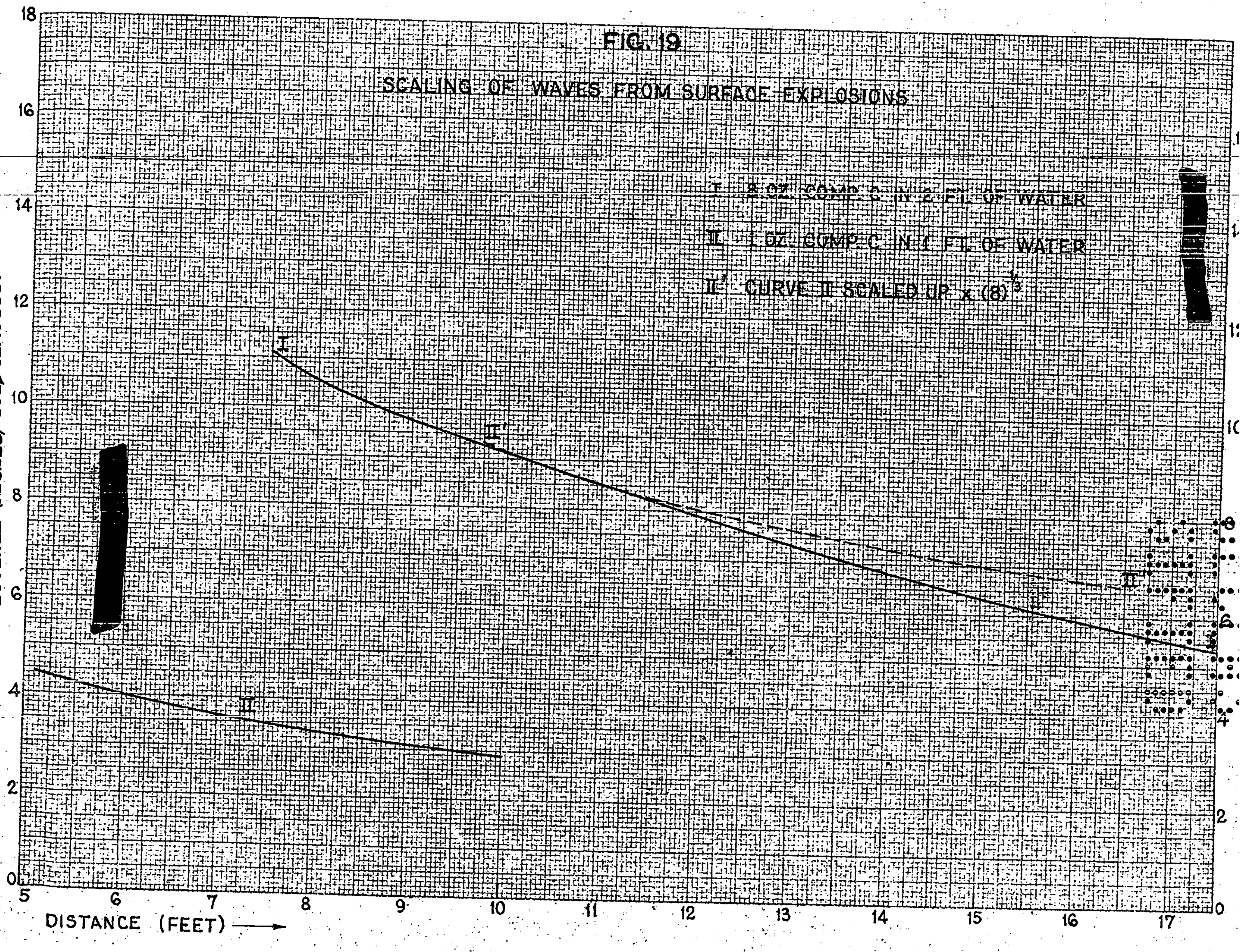
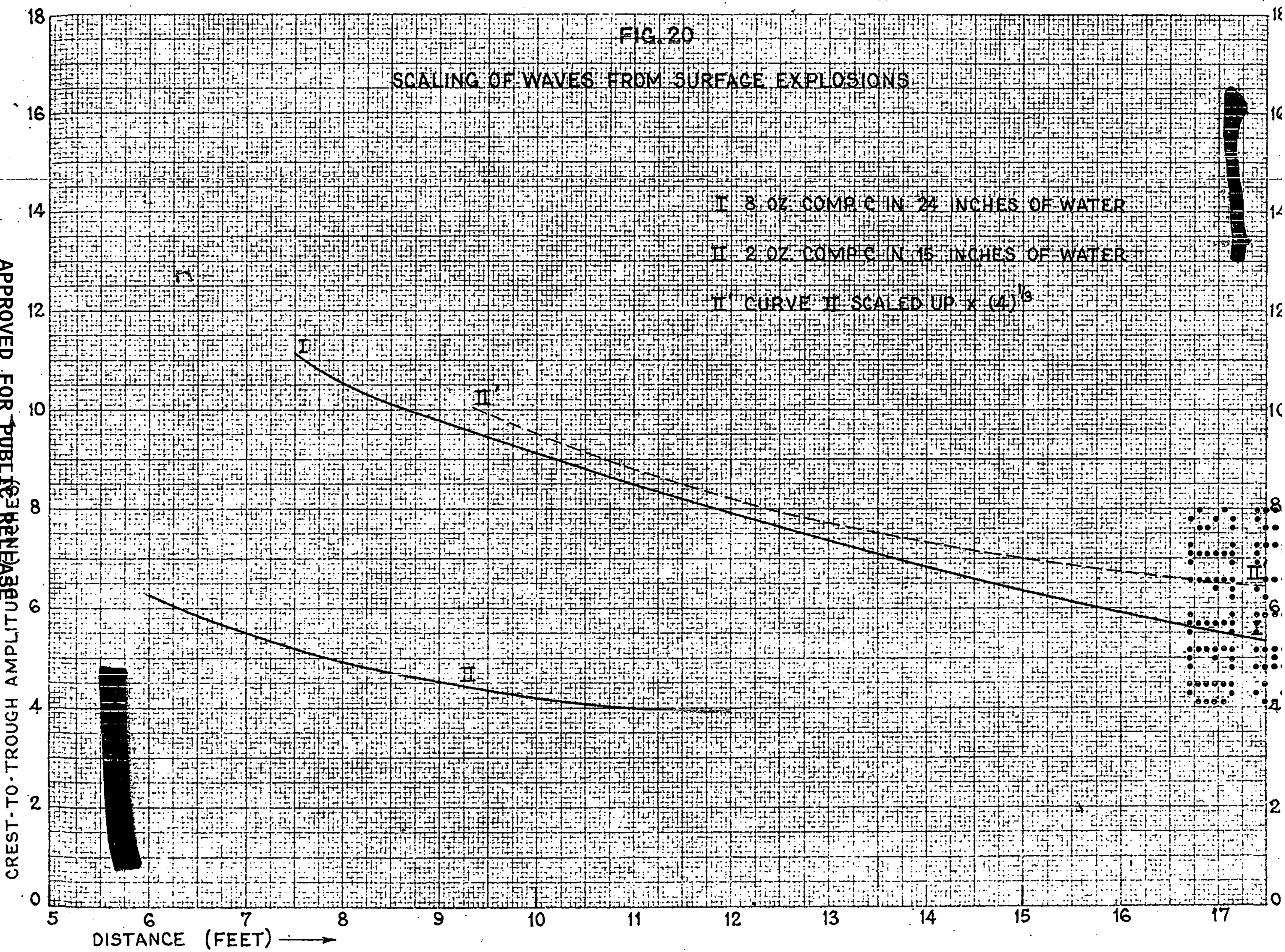


FIG. 19
SCALING OF WAVES FROM SURFACE EXPLOSIONS
II 1 OZ. COMP. C IN 1 FT. OF WATER
II' CURVE II SCALED UP x (8)^1/3
I
II'
II
CREST-TO-TROUGH AMPLITUDE
DISTANCE (FEET)
0
2
4
6
8
10
12
14
16
18
5
6
7
8
9
10
11
12
13
14
15
16
17



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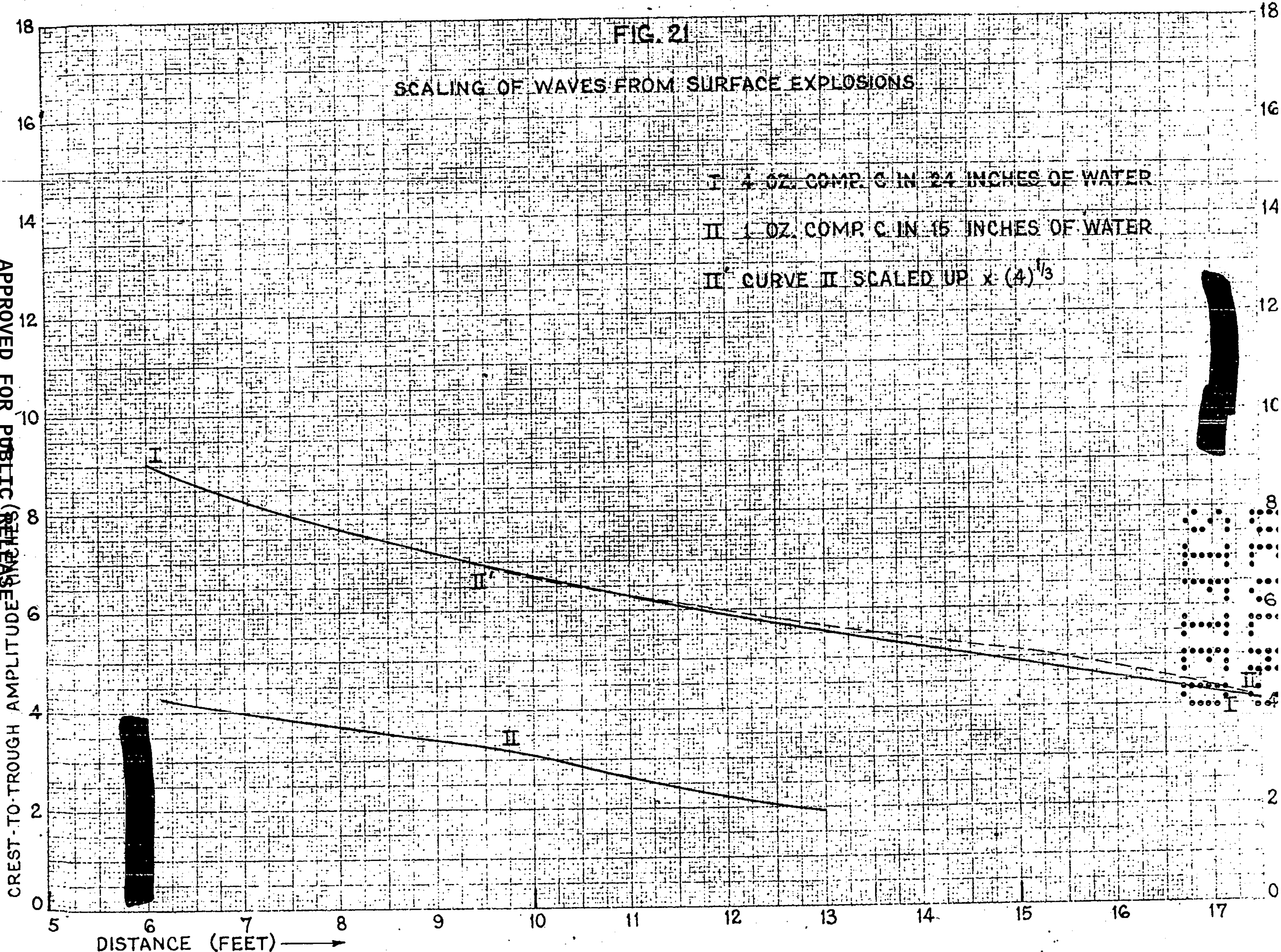

FIG. 21

SCALING OF WAVES FROM SURFACE EXPLOSIONS

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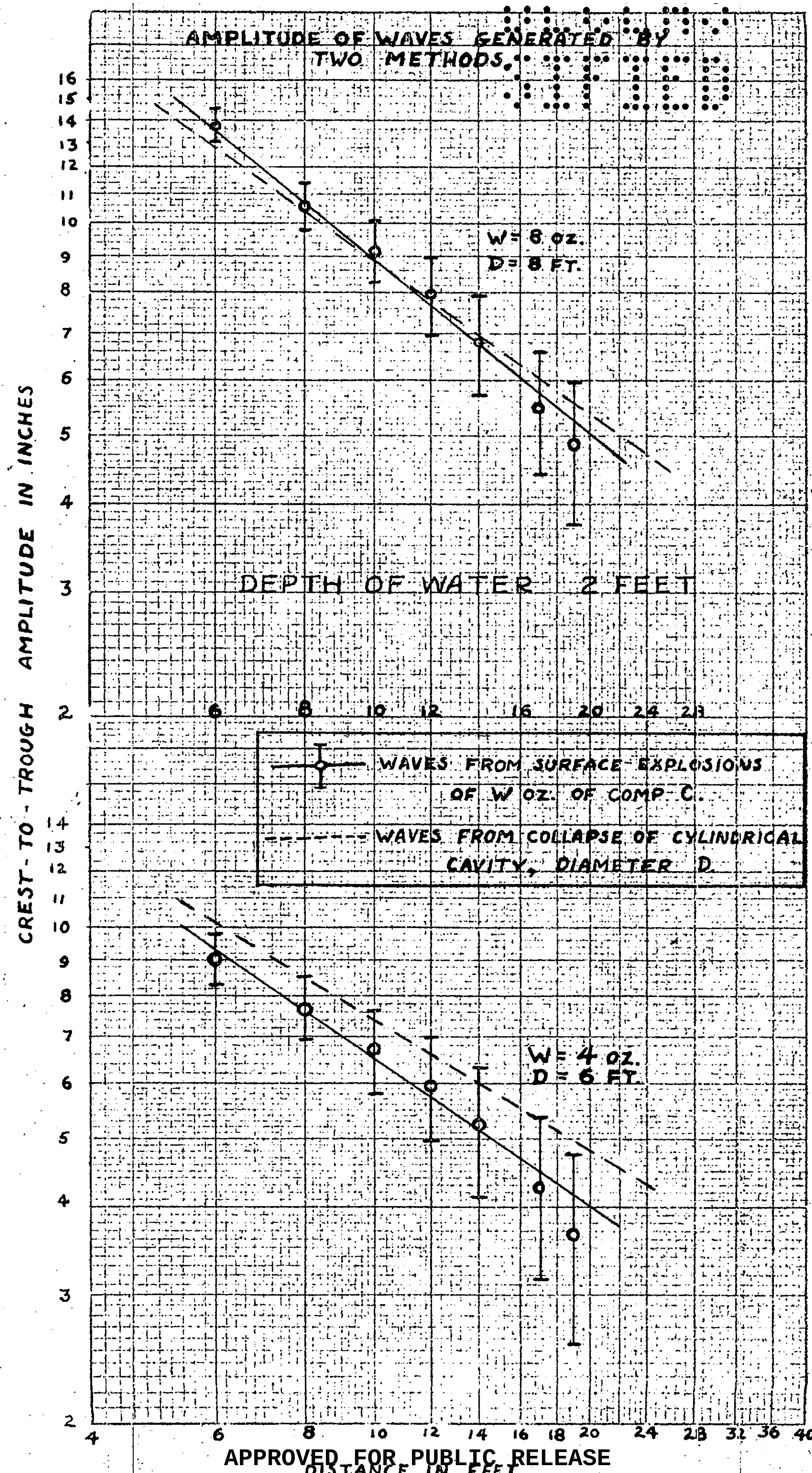

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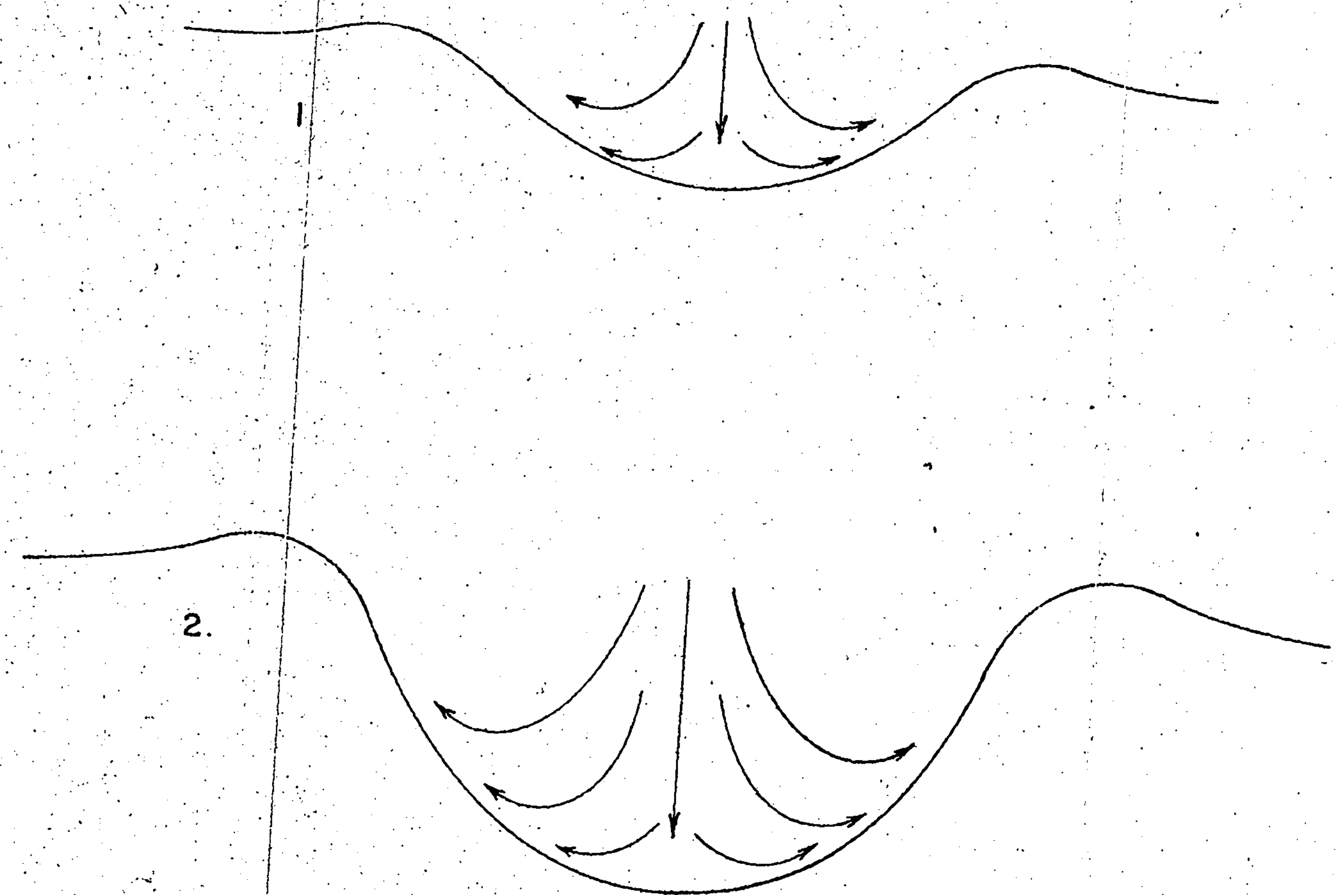
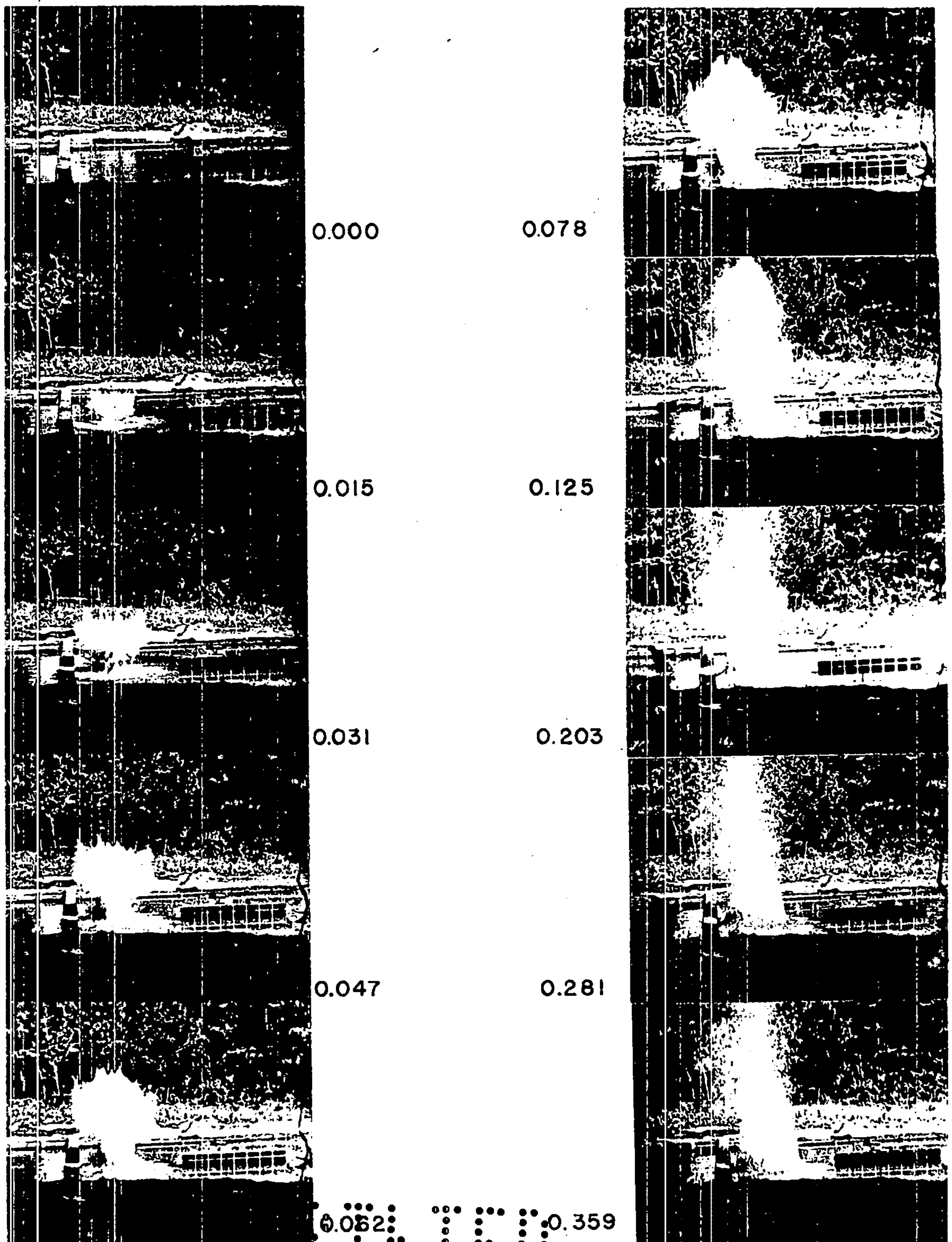


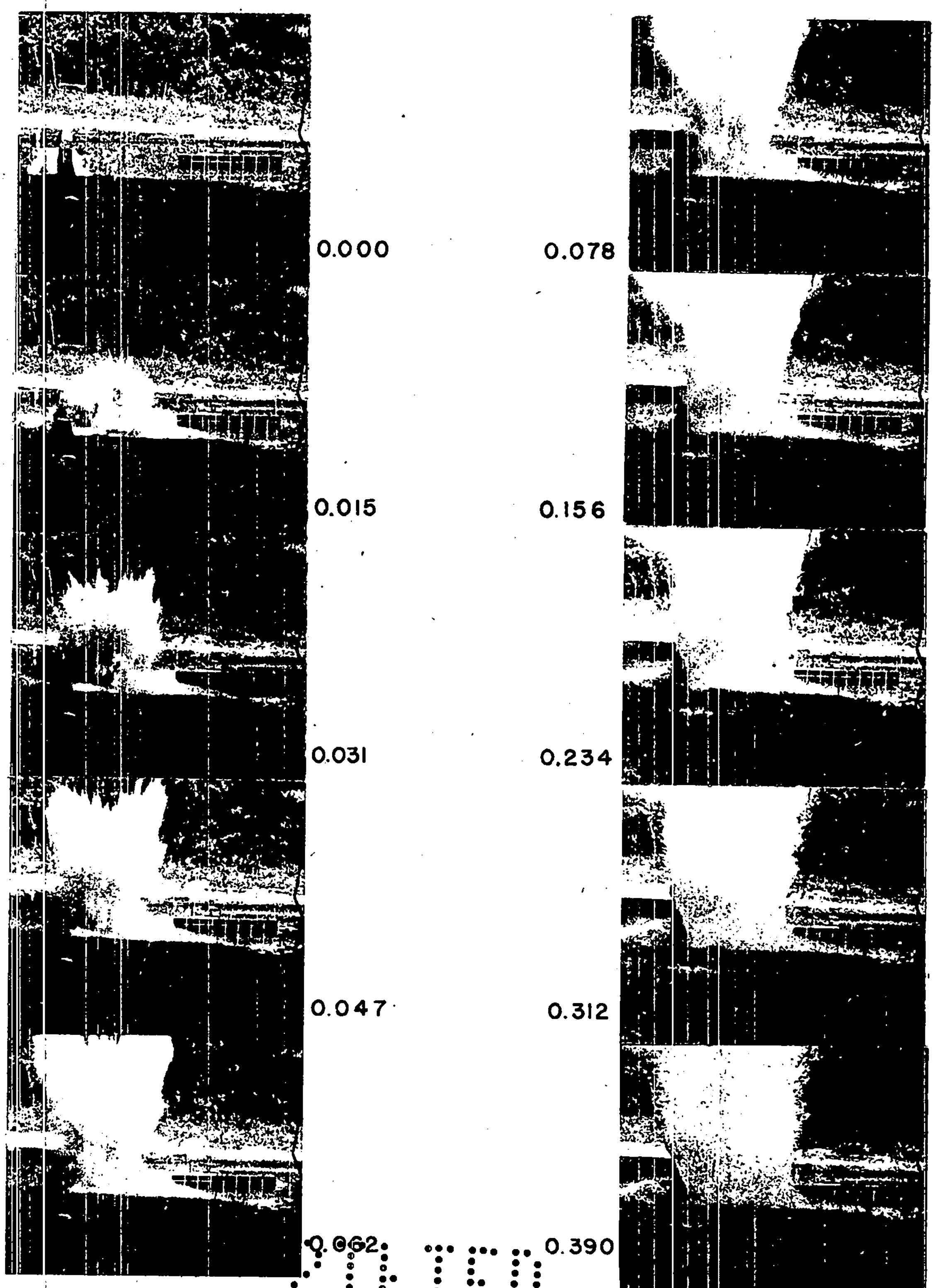
FIG. 23. TWO HYPOTHETICAL STAGES IN THE CREATION OF A CAVITY BY A SURFACE EXPLOSION IN WATER.

FIG. 24. EXPLOSION OF 1 OZ. COMP C
AT THE SURFACE OF WATER 2 FT. DEEP

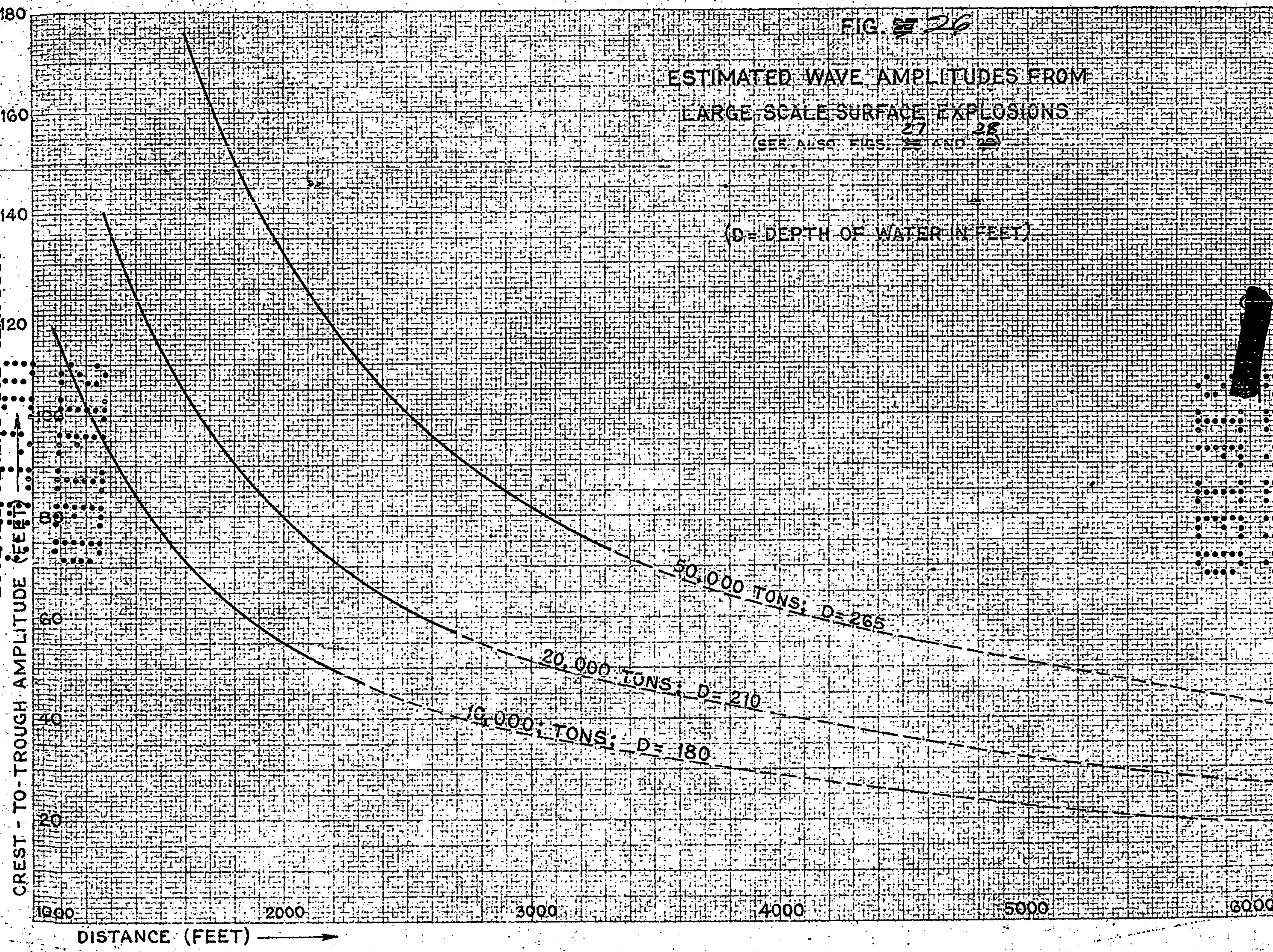


TIME IN SECONDS

FIG. 25. EXPLOSION OF 8 OZ. COMP. C
AT THE SURFACE OF WATER 2 FT. DEEP



TIME IN SECONDS



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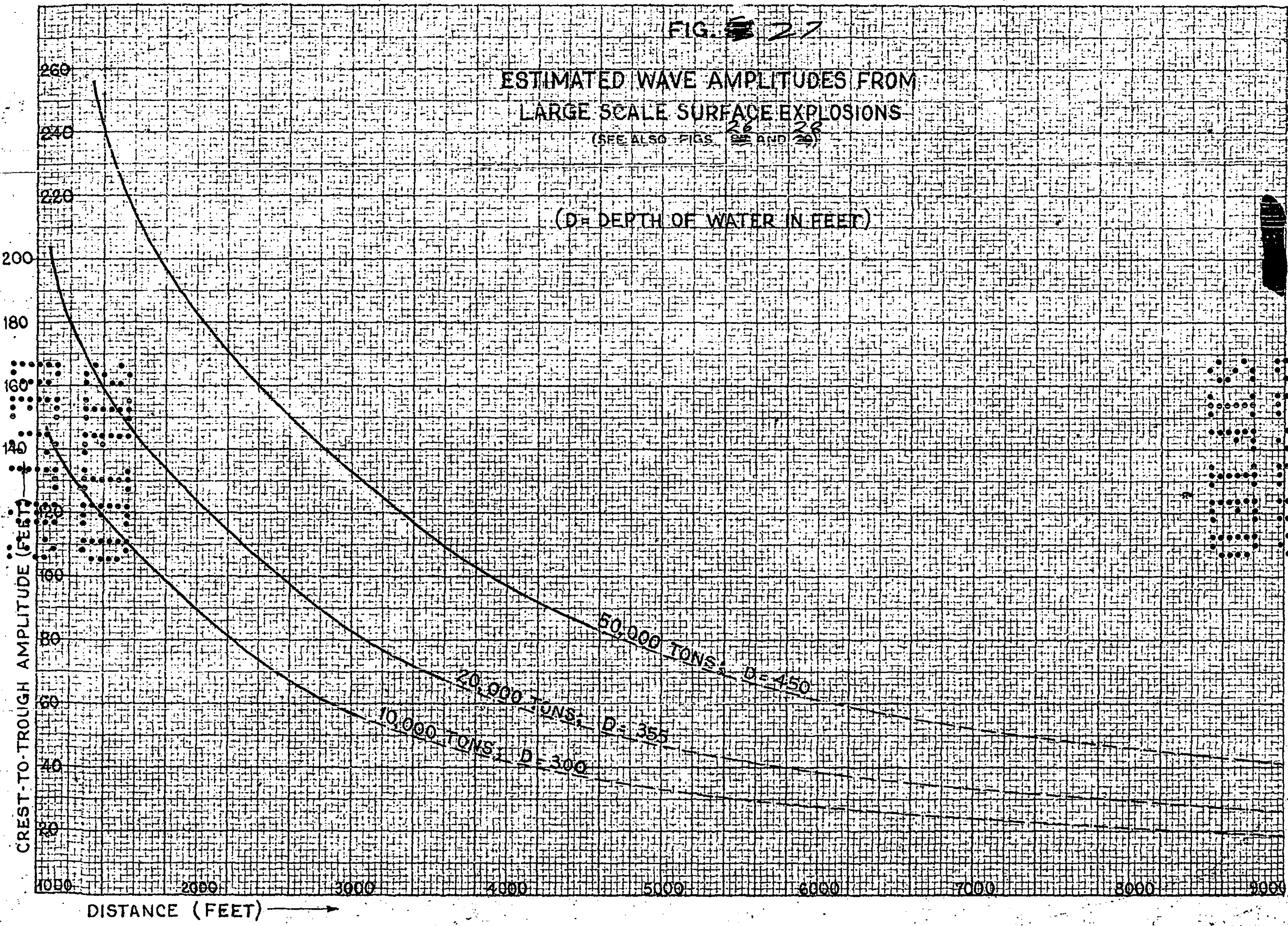

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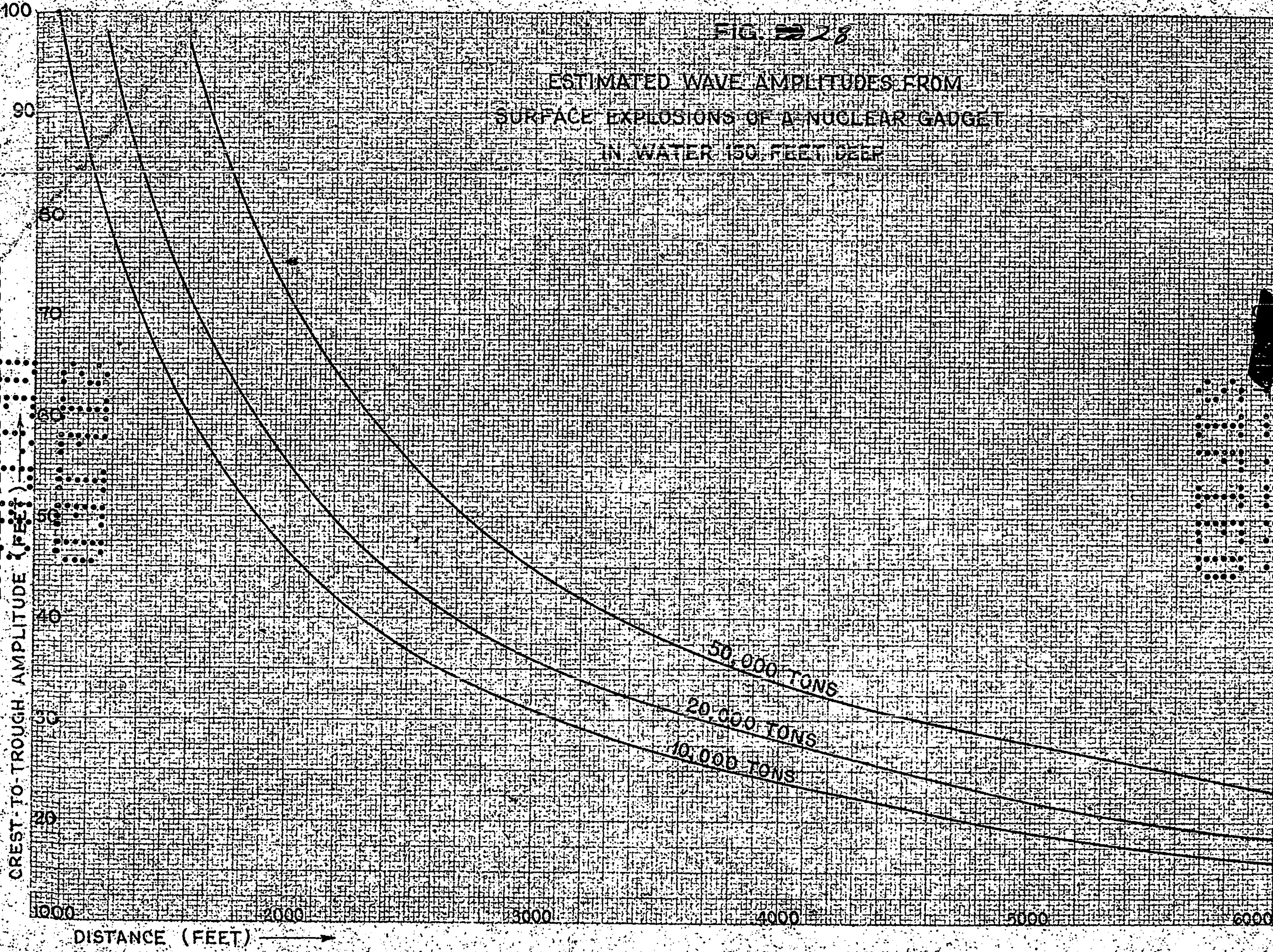

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